

基于计算拓扑的拓扑理解和拓扑控制

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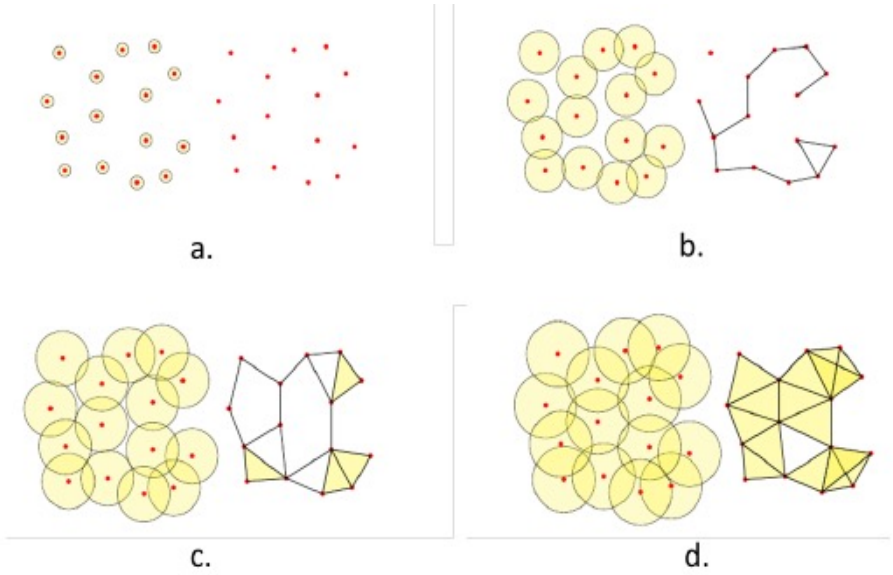
01

持续同调简介

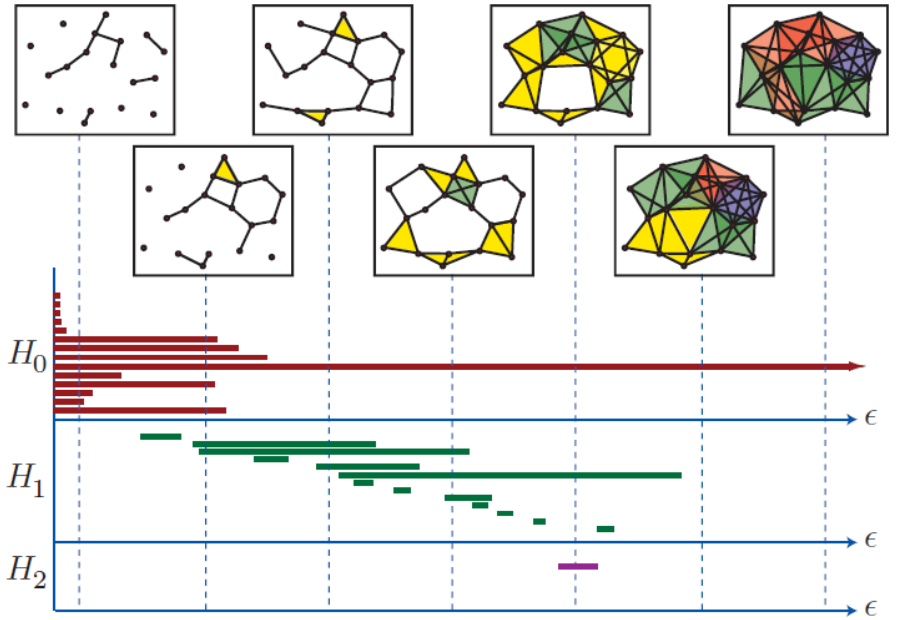


Filtration，Persistent Homology



Filtration (Čech complex)

As the radii of open balls grow, the **simplicial complex** can be constructed on point cloud data, which gives a nested complex sequence, a **filtration**.



Barcode

- As time goes by, the radii of open balls grow
- New *k*-holes are **born**
- Some *k*-holes are destroyed by higher-dimensional simplex and **die**

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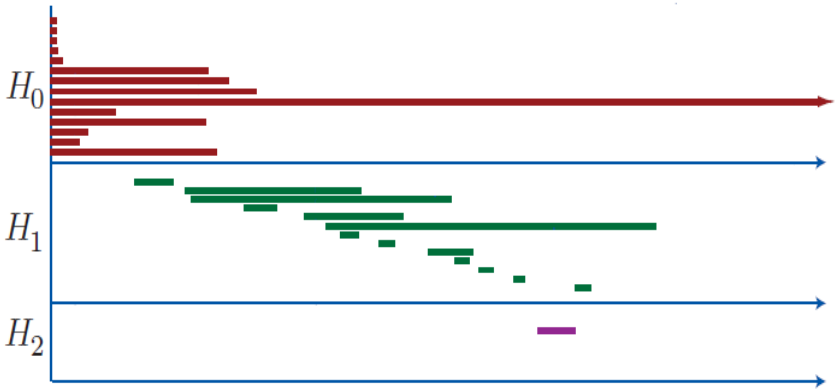
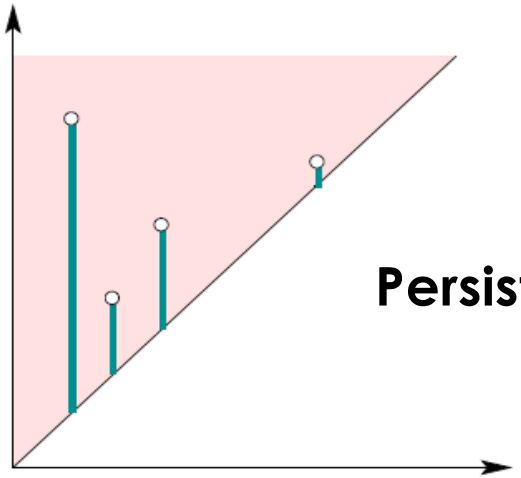
持续同调（Persistent Homology）



Barcode(条码)



Persistence Diagram (持续图)

已经建立了比较完整的理论体系：

- Distance definition and computation
- Stability
- Representation
- Extended persistence
- Zigzag persistence
- Multiparameter persistence
- Persistence Module
- Quiver

…

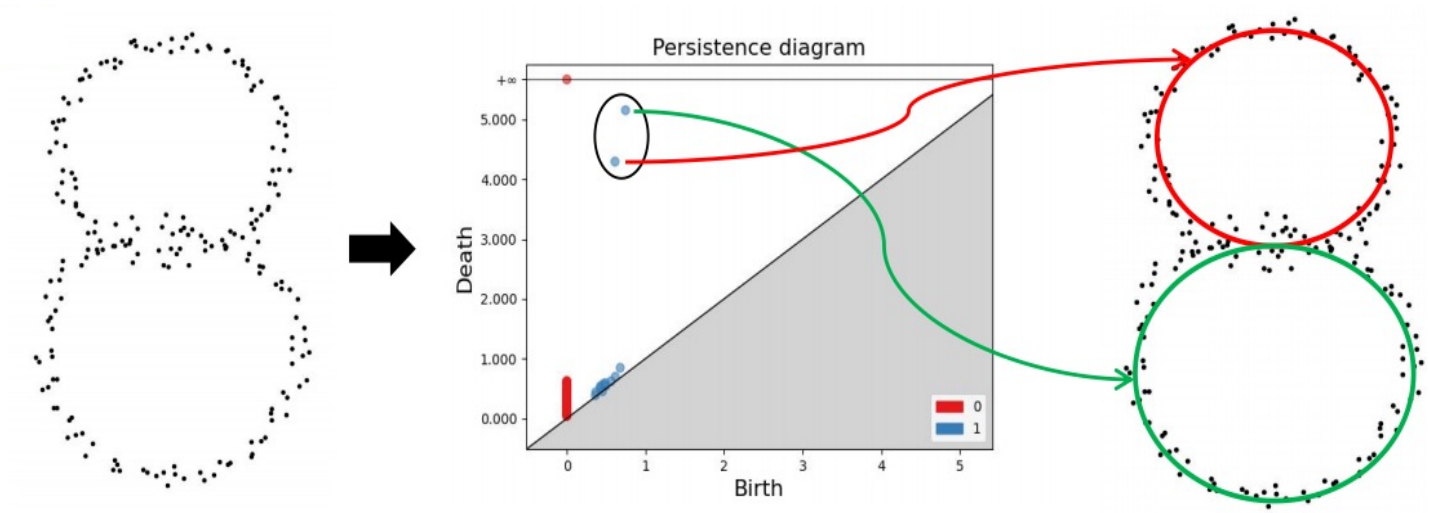
02

同调代表元



同调代表元

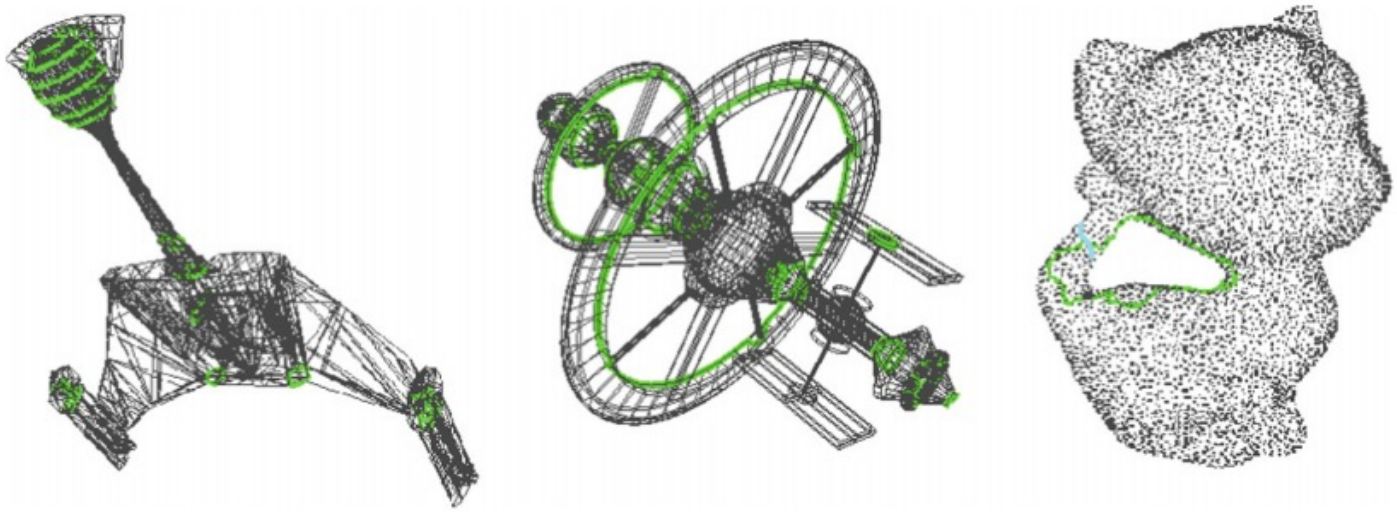
- 在许多问题中，我们不仅想要研究几何对象里有多少个显著拓扑特征，还希望找到这些拓扑特征的具体的几何表示
- 同调代表元就提供了这样的功能：它是同调群的生成元的某个具体的代表元，这些代表元可以具体地刻画出几何对象（如点云）里面所存在的“洞”的特征
- 对于单纯复形上的同调代表元，1维代表元可以表示为一个分段线性闭曲线，2维代表元可以表示为一个闭三角曲面，等等



- 持续同调也提供了对应的同调代表元：对于一个持续配对(b,d)，
  - 若$d \neq +\infty$，则对应的持续闭链(persistent cycle)是K_b中包含σ_b的闭链，它在K_{d-1}中不是边缘，但在K_d中成为边缘
  - 若 $d = +\infty$，则是K_b中包含σ_b的闭链
  - 持续闭链通常不是唯一的

最小持续1-闭链

- 最小持续1-闭链：
- 若给每条边（1-单形）赋予非负权重，1-闭链的权重定义为其包含所有边的权重之和，则最小持续1-闭链是该区间对应的所有可能的持续1-闭链中权重最小的闭链，它就是一种基于权重最小条件筛选出来的拓扑代表元



Dey T K, Hou T, Mandal S. Persistent 1-cycles: Definition, computation, and its application[C]//International workshop on computational topology in image context. Cham: Springer International Publishing, 2018: 123-136.

体积最优闭链

- 体积最优闭链：
  - 以1维的代表元为例，在过滤的过程中，一个持续1-闭链首先在某个时间出现，然后被另一个2维的链填满（即变成这个2维链的边界）
  - 因此，可以先求出这个2维链，然后求其边界就能得到对应的持续1-闭链（称之为体积最优1-闭链）
- 这个2维链以及对应的持续1-闭链可以通过求解一个线性规划问题得到：即先找到一个对应于(b,d)的负2-单形σ_d，然后通过线性规划找到包含σ_d且满足其他约束条件一个2-链，最后求它的边界得到体积最优1-闭链
- 这种方法也适用于求解任何维度的代表元，因而有很广泛的适用性

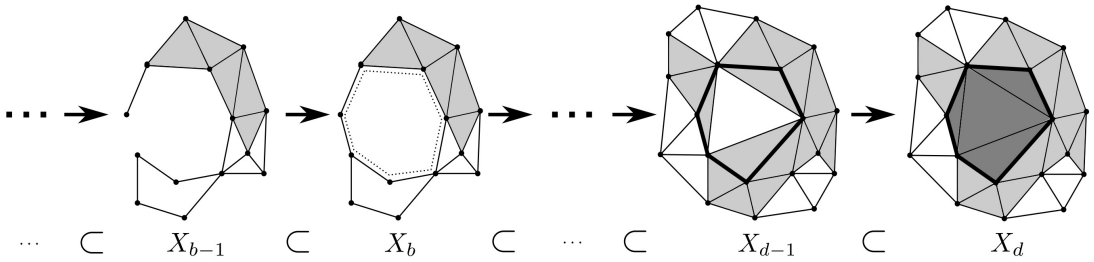


Figure 3. *Optimal cycle and volume-optimal cycle for a simplicial filtration. In this filtration a homology generator appears at $H_1(X_b)$ and the generator disappears at $H_1(X_d)$. The dotted loop on X_b is the optimal cycle, the dark gray 2-chain (triangles) on X_d is the optimal volume, and the boundary of the optimal volume (the thick line on X_d) is the volume-optimal cycle.*

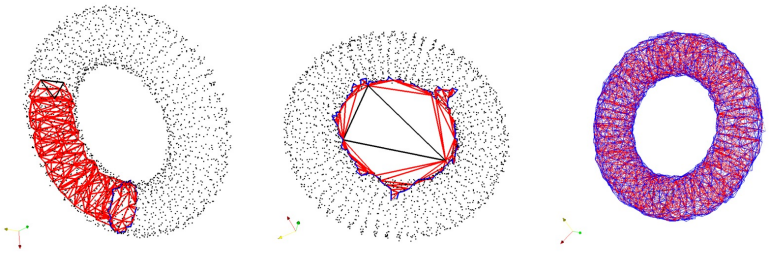


Figure 15. *Volume-optimal cycles for $(0.001, 0.072)$ and $(0.001, 0.453)$ in D_1 and $(0.008, 0.081)$ in D_2 on the torus point cloud.*

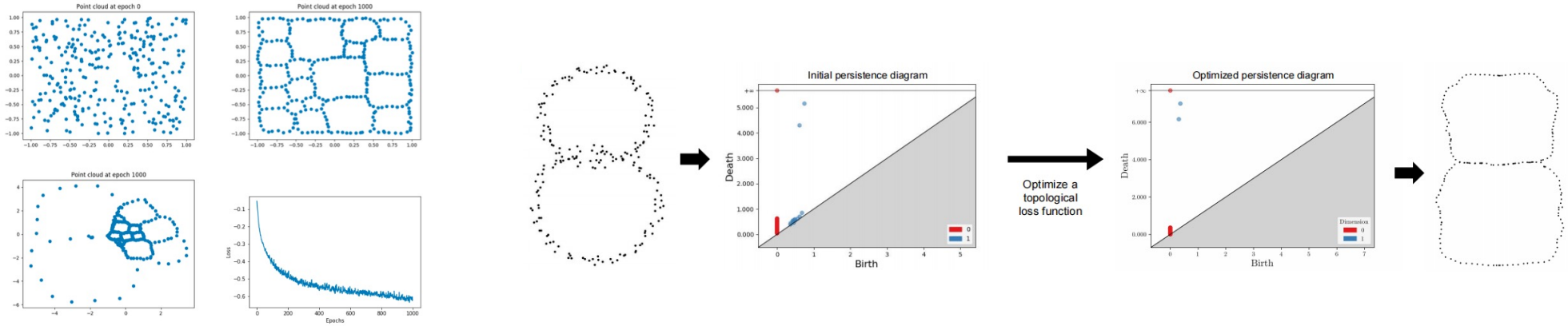
求是创新 Obayashi I. Volume-optimal cycle: Tightest representative cycle of a generator in persistent homology[J]. SIAM Journal on Applied Algebra and Geometry, 2018, 2(4): 508-534.

03
基于持续图的优化



拓扑控制的理论基础：持续图的优化

- 拓扑控制：在几何设计中，我们通常希望模型满足特定的拓扑性质
  - 几何设计领域缺乏相关理论和方法
  - 难点：拓扑量（如 Betti 数）通常难以直接微分
- 已经证明，定义在持续图上的函数 $L(X) = l(Dgm(X))$，其中 $Dgm(X)$ 是模型X 的持续图，在广泛的情况下是可微的，从而可以通过优化方法控制模型X的拓扑结构



为几何模型的拓扑控制带来了系统理论和方法，对于几何设计具有重要意义

Carrière, M., Chazal, F., Glisse, M., Ike, Y., Kannan, H., & Umeda, Y. (2021). Optimizing persistent homology based functions. In M. Meila & T. Zhang (Eds.), *Proceedings of the 38th International Conference on Machine Learning* (Vol. 139, pp. 1294–1303). PMLR.

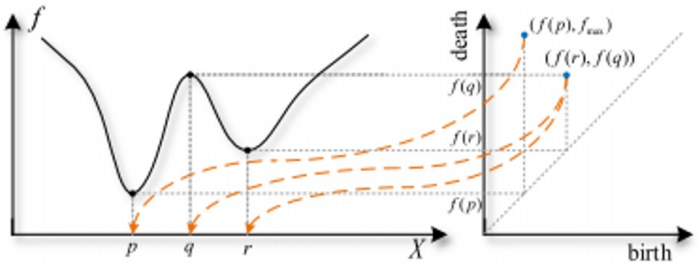
求是创新

可微函数的构造

- 设L是定义在持续图上的函数，若 L 为局部Lipschitz函数，则在某些一般性假设下， L可微且随机梯度下降算法收敛
- PyTorch等标准库中提供的随机梯度下降算法可用于持续图的求梯度计算
- 例子：下述函数（持续度求和、两个PD的瓶颈距离）都是可以求梯度的优化目标函数：

$$E(D) = \sum_{i=1}^{p} |d_i - b_i|$$

$$E(D) = d_B(D_{\text{reg}}, D^*_{\text{reg}}) = \min_{m} \max_{(p,p^*)\in m} \|p - p^*\|_\infty$$

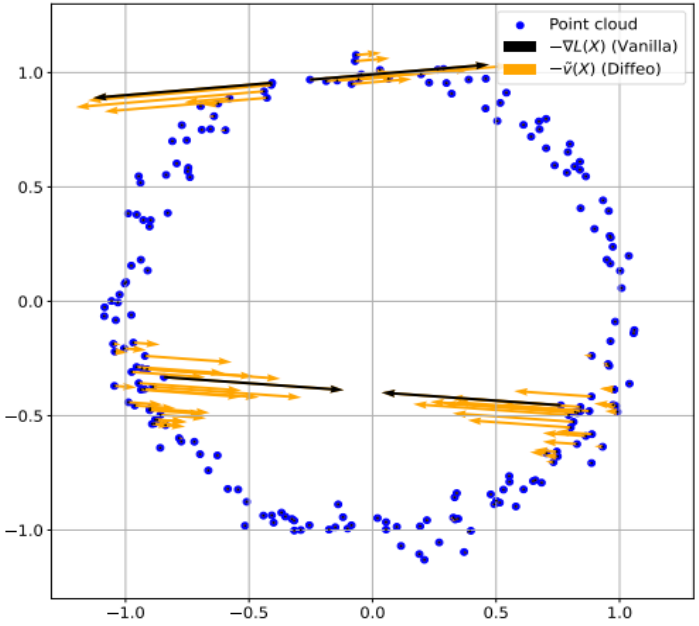


(b) An example of topological inverse mapping.

Carriere M, Chazal F, Glisse M, et al. Optimizing persistent homology based functions[C]//International conference on machine learning. PMLR, 2021: 1294-1303.

稀疏优化方法

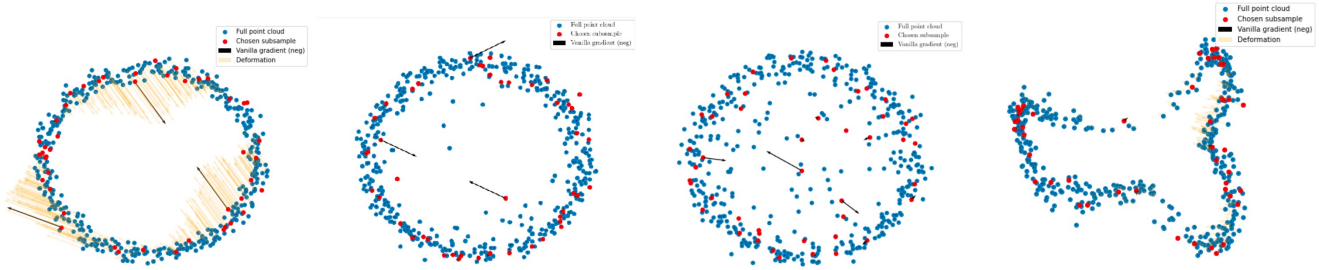
- 求梯度过程中可能会出现稀疏梯度 (Sparse Gradients) 的情况：传统的拓扑梯度可能非常稀疏
  - 以对点云的优化为例，对于一个包含 n 个点的点云，仅对点云中的极少数点求梯度时的梯度非零
  - 这导致大部分点在迭代中不移动，导致优化效率极低，且容易陷入不良局部最优解
- 图中的例子就表明，传统的求梯度优化方法仅在4个点上（黑色箭头）有非0梯度



Carriere M, Theveneau M, Lacombe T. Diffeomorphic interpolation for efficient persistence-based topological optimization[J]. Advances in Neural Information Processing Systems, 2024, 37: 27274-27294.

稀疏优化方法

- 解决方案：将稀疏的拓扑梯度 $\nabla L(X)$ 视为一组离散的向量，利用再生核希尔伯特空间将其插值及平滑化，生成定义在全空间 $\mathbb{R}^d$ 上的平滑向量场
- 这样就可以使待优化的点云中的所有点都会协同移动，加速优化结果的收敛；同时，可以保证这样的变换是微分同胚的，从而保持了数据的局部几何结构
- 下图中的黄色箭头说明通过可微向量场插值的方法可以使更多点上都有非0梯度



(a) $t = 0$ (b) $t = 100$ (Vanilla) (c) $t = 500$ (Vanilla) (d) $t = 100$ (Diffeo)

Figure 3: Showcase of the usefulness of subsampling combined with diffeomorphic interpolations to minimize a topological simplification loss, with parameters $\lambda = 0.1$, $s = 50$, $n = 500$. (a) Initial point cloud X (blue), subsample X' (red), vanilla topological gradient on the subsample (black) and corresponding diffeomorphic interpolation (orange). (b) and (c), the point cloud X_t after running $t = 100$ and $t = 500$ steps of vanilla gradient descent. (d) the point cloud X_t after running $t = 100$ steps of diffeomorphic gradient descent.

求是创新 Carriere M, Theveneau M, Lacombe T. Diffeomorphic interpolation for efficient persistence-based topological optimization[J]. Advances in Neural Information Processing Systems, 2024, 37: 27274-27294.

稀疏优化方法

- 由于计算全量数据的持续图太慢，上述方法还可以结合子采样（Subsampling）的策略 。
- 具体做法为：从大数据集 X 中随机采样一个小批次 X' ，然后在 X' 上计算稀疏拓扑梯度。利用上述微分同胚插值，将 X' 上的梯度推广到全空间向量场 V 上，接着就可以使用向量场 V 更新原始的大数据集 X。
- 上述操作使得拓扑函数优化可以应用在数万个点的数据集上，这在传统方法中是不可行的 。

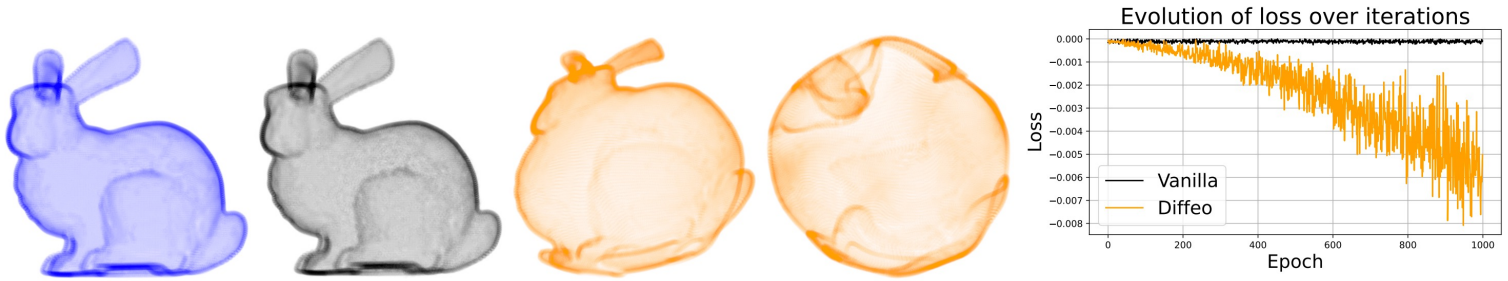


Figure 5: From left to right: initial Stanford bunny X_0, the point cloud after $1,000$ epochs of vanilla topological gradient descent (barely any changes), the point cloud after 200 epochs of diffeomorphic gradient descent, after 1,000 epochs, and eventually the evolution of losses for both methods over iterations.

Carriere M, Theveneau M, Lacombe T. Diffeomorphic interpolation for efficient persistence-based topological optimization[J]. Advances in Neural Information Processing Systems, 2024, 37: 27274-27294.

使用出生和消亡上链的拓扑优化

- 在经典持续同调计算中，出生和消亡单纯形具有高度局部性，并且通常无法在数据集的扰动下保持稳定
- 一种新方法是引入出生和消亡上链(cochain)为单纯形的线性组合，定义为一个 l_2 优化问题的解
  - 对于给定的持续上同调类，这些上链始终是唯一的

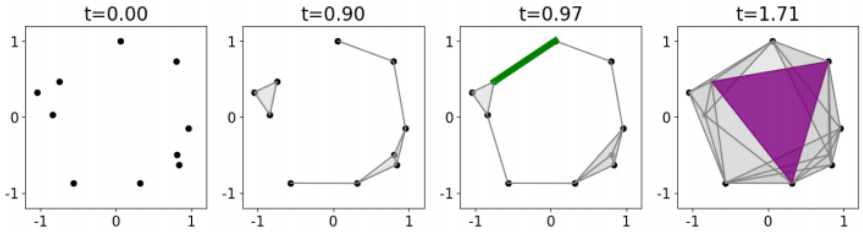


Figure 1: The classical notion of birth simplex (in green) and death simplex (purple) for degree-1 persistent homology.

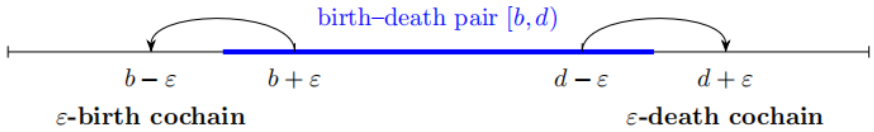


Figure 2: The ε-birth cochain which lives at $b-\varepsilon$ is computed for a cochain at $b+\varepsilon$, and the ε-death cochain which lives at $d+\varepsilon$ is computed for a cochain at $d-\varepsilon$. See Subsection 3.2.

- 进一步，基于出生和消亡上链定义了对应的出生与消亡含量(Birth and Death Content)，用它们代替了传统拓扑优化函数里面使用的出生和消亡时间
- 采用这些出生与消亡含量的损失函数可以得到效果更好的优化结果

求是创新 Weighill T, Zhou L. Topological optimization with birth and death cochains[J]. arXiv preprint arXiv:2603.25575, 2026.

使用出生和消亡上链的拓扑优化

浙江大学 ZHEJIANG UNIVERSITY

- 基于出生与消亡含量的目标函数相较于传统方法有如下主要优点：

a) 梯度连续性：在传统方法中，点云的微小移动可能会导致正单形（产生拓扑特征的单形）或者负单形（消灭拓扑特征的单形）发生突变（例如从一个单纯形变成另一个单纯形 ），这可能导致损失函数的梯度不连续甚至不存在；而出生与消亡含量是基于上链的线性组合的，其系数随几何位置连续变化，从而保证了梯度的平滑性和连续性

b) 唯一性：在很多几何构型中，由于同调代表元不唯一，其对应的正负单纯形也可能不唯一的；但通过 l_2 最小化解出的出生/消亡上链在数学上是唯一确定的，这为优化算法提供了明确、唯一的搜索方向

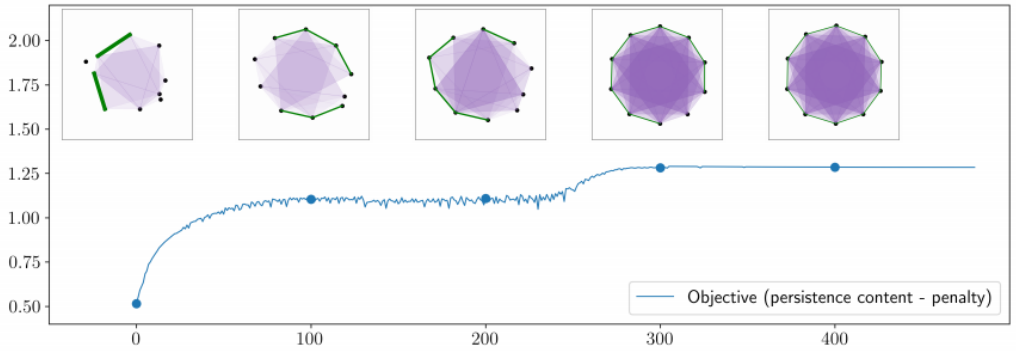


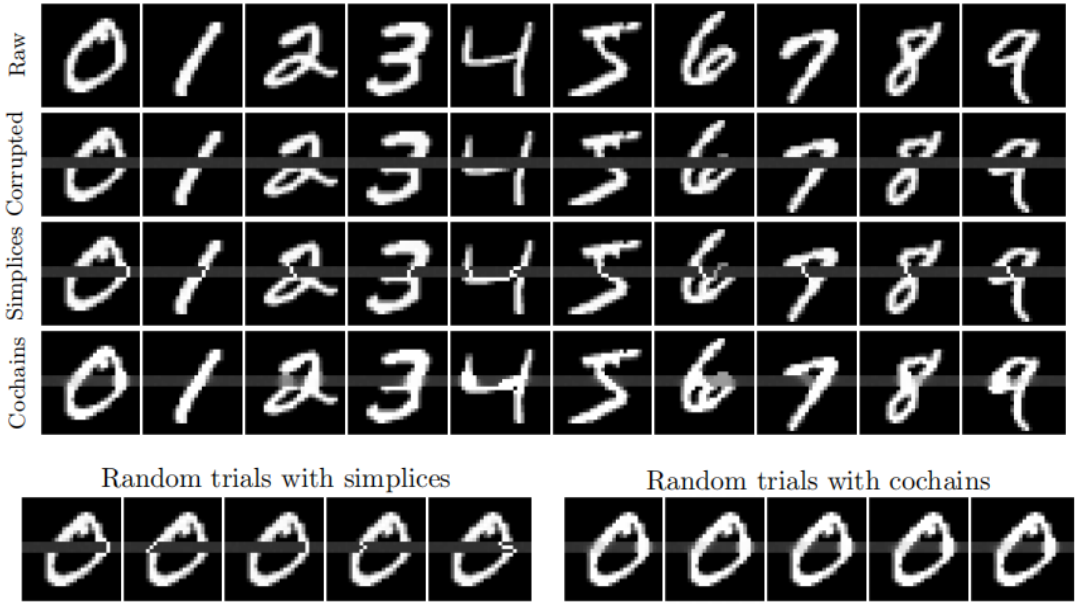
Figure 3: Optimizing a point cloud using birth and death cochains (see Section 7). We plot the objective function to be maximized, along with snapshots of the points cloud every 100 iterations. The birth cochain is shown using green edges and the death cochain is shown using purple triangles; these are the simplices that are adjusted in the gradient ascent step.

Weighill T, Zhou L. Topological optimization with birth and death cochains[J]. arXiv preprint arXiv:2603.25575, 2026.

求是创新

使用出生和消亡上链的拓扑优化

- 实验验证表明，使用出生和消亡上链来代替传统方法，在修复被墨迹污染的MNIST数据集上的表现更好
- 这项研究还在北极冰盖卫星图像数据集上做了拓扑特征提取实验，发现出生和消亡上链的方法比传统方法可以更加鲁棒地提取出真实图像中存在的拓扑特征



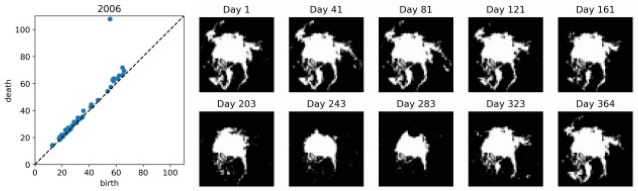


Figure 13: The degree-1 persistence diagram for Arctic ice data from 2006 (left) and snapshots of that data (right).

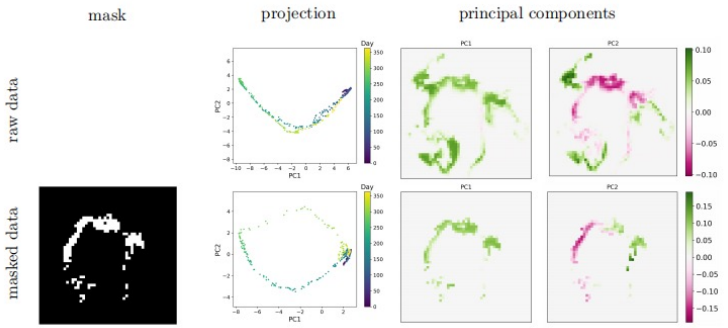


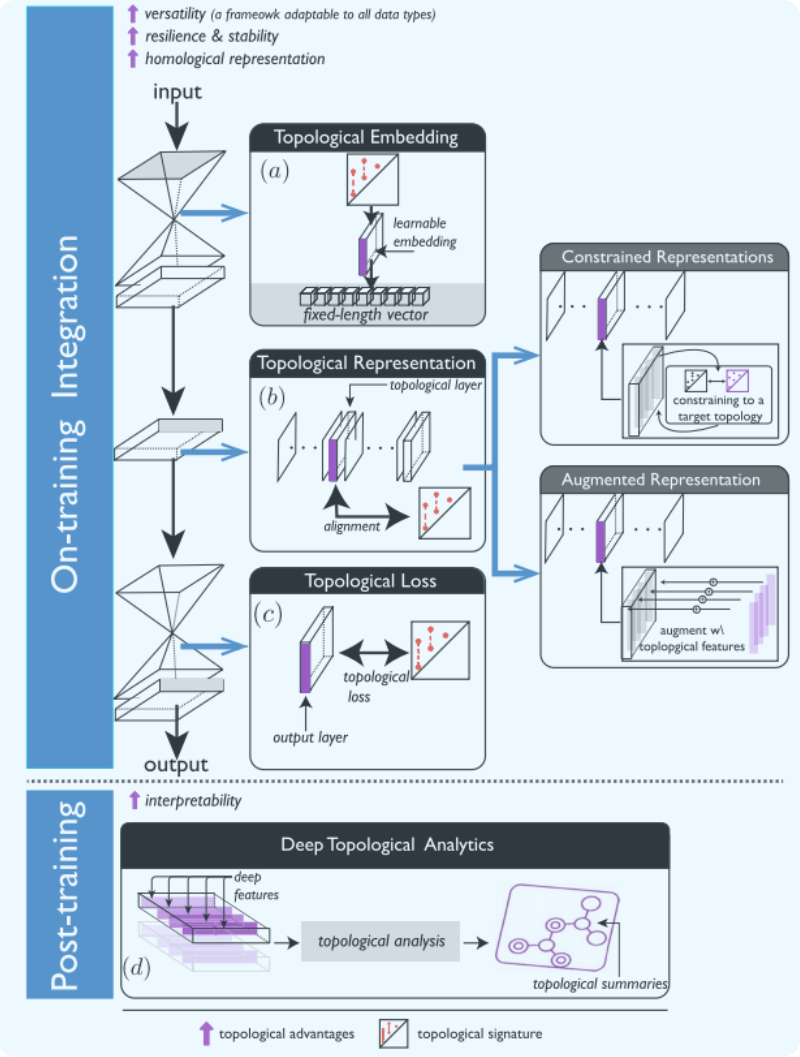
Figure 14: Feature selection to reveal and explain degree-1 persistence in the 2006 data. We show the mask obtained by selecting pixels based on their gradients. We then show PCA plots for the raw data and masked data. The scatter plots show the projection onto the principal components. Each principal component is visualized as a heat map on the right.

求是创新 Weighill T, Zhou L. Topological optimization with birth and death cochains[J]. arXiv preprint arXiv:2603.25575, 2026.

04
拓扑深度学习



拓扑深度学习的三种常见范式



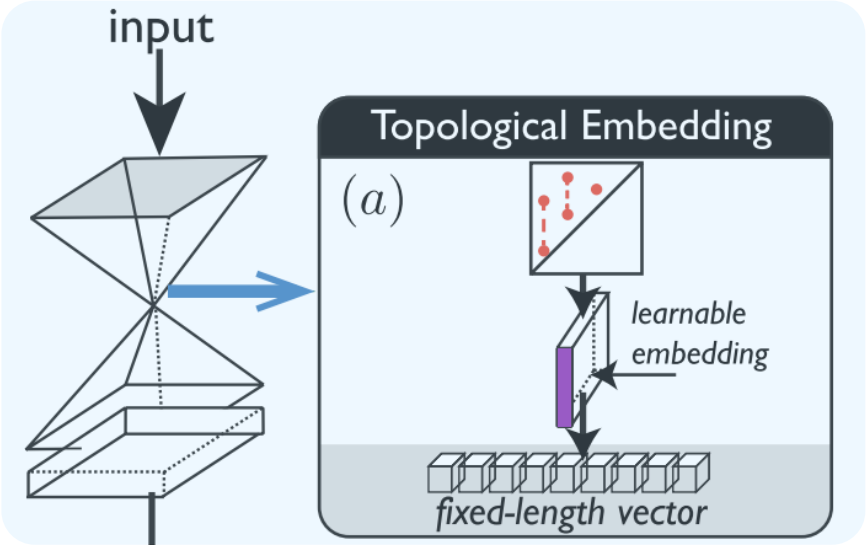
- 拓扑深度学习（Topological Deep Learning)的思想是将拓扑数据分析的方法（如持续同调）和深度学习相结合，用拓扑信息来增强深度学习的效果
- 拓扑不是来“替代”深度学习的，而是来补全它缺失的几何与全局视角
- 常见的拓扑深度学习有下述三种范式：

1. 将拓扑信息作为先验，和原始数据一起作为神经网络的输入
2. 在网络中设计一个拓扑层，用于处理拓扑特征
3. 在损失函数中加入拓扑损失作为约束

Zia A, Khamis A, Nichols J, et al. Topological deep learning: a review of an emerging paradigm[J]. Artificial Intelligence Review, 2024, 57(4): 77.

拓扑先验作为深度学习输入

- 第一种方式是让拓扑信息（比如持续图）像图像或者点云一样，成为模型的输入之一
- 由于持续图是以点对来表示的，因此不能直接作为神经网络的输入
- 因此，要先把持续图做特征嵌入，即把持续图进行向量化，转化为可以输入神经网络的形式

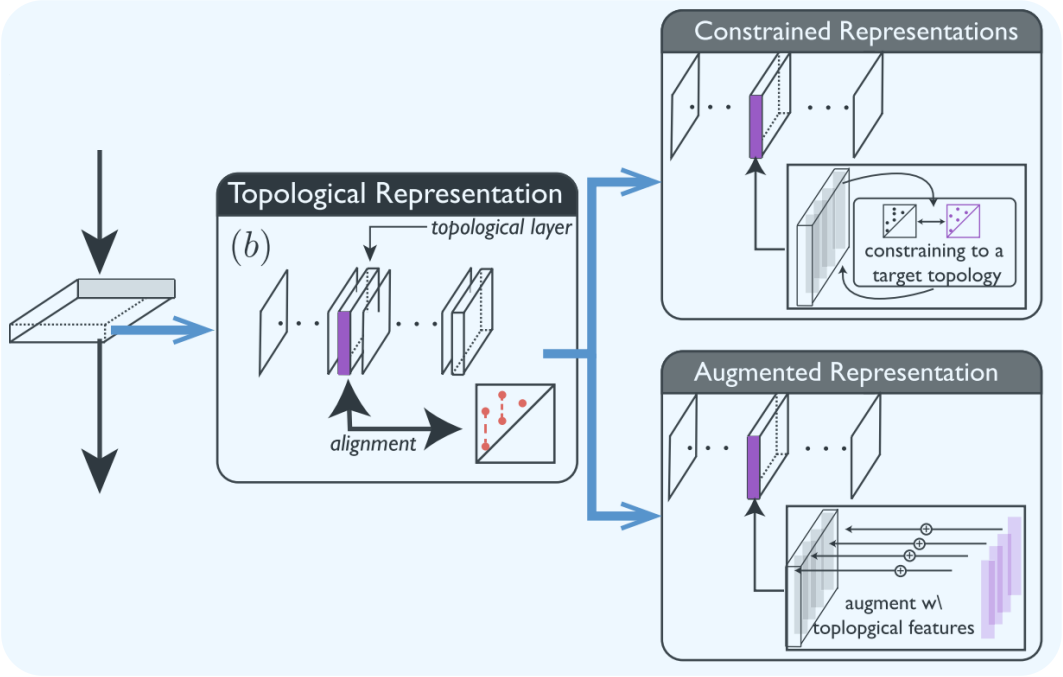


- 常见的向量化方式有持续图像(Persistence Image, PI)，持续景观(Persistence Landscape, PL)，持续B样条（Persistence B-spline, PB)等
- 这些特征较好地保留了持续图里面原始的拓扑信息，因而可以为网络提供额外的拓扑先验，可以在特定的任务（特别是与拓扑有关的）上面起到较好的效果

在网络中增加拓扑层

- 第二种方式是在网络中插入可学习的拓扑层，处理拓扑特征（如PD、PL等）
- 拓扑层就是像CNN、Attention层一样的一个网络的中间层，它一般设计为如下功能的层：

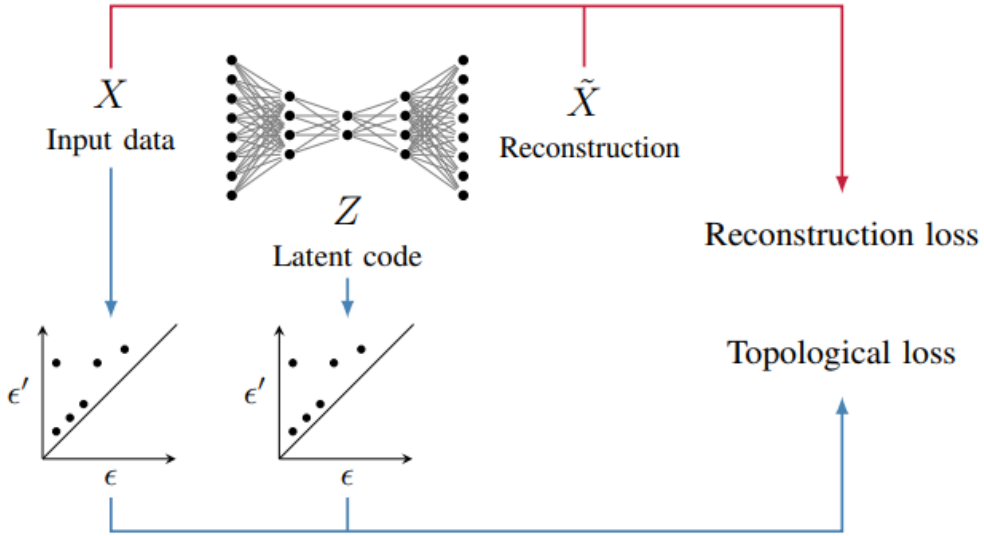
  输入张量 → 构造过滤 → 计算拓扑特征 (PD/PI/PL) → 可微向量化 → 下游张量



求是创新

在网络中增加拓扑层

- 例：Topological Autoencoders
- 这个拓扑层把隐空间 Z 视为点云，学习一个 $f_\theta(z_i)$ 当“半径”，做 Rips过滤，使得计算结果能够最小化输入的 PD与隐空间的 PD 距离

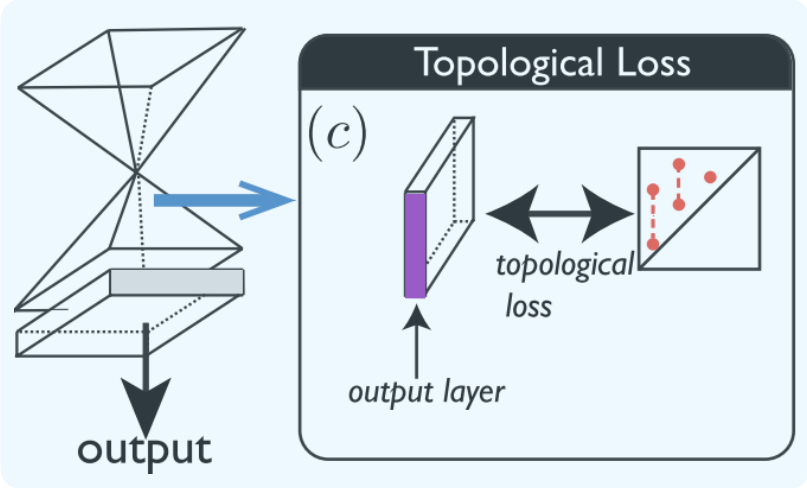


Moor M, Horn M, Rieck B, et al. Topological autoencoders[C]//International conference on machine learning. PMLR, 2020: 7045-7054.

将拓扑融入损失函数的设计

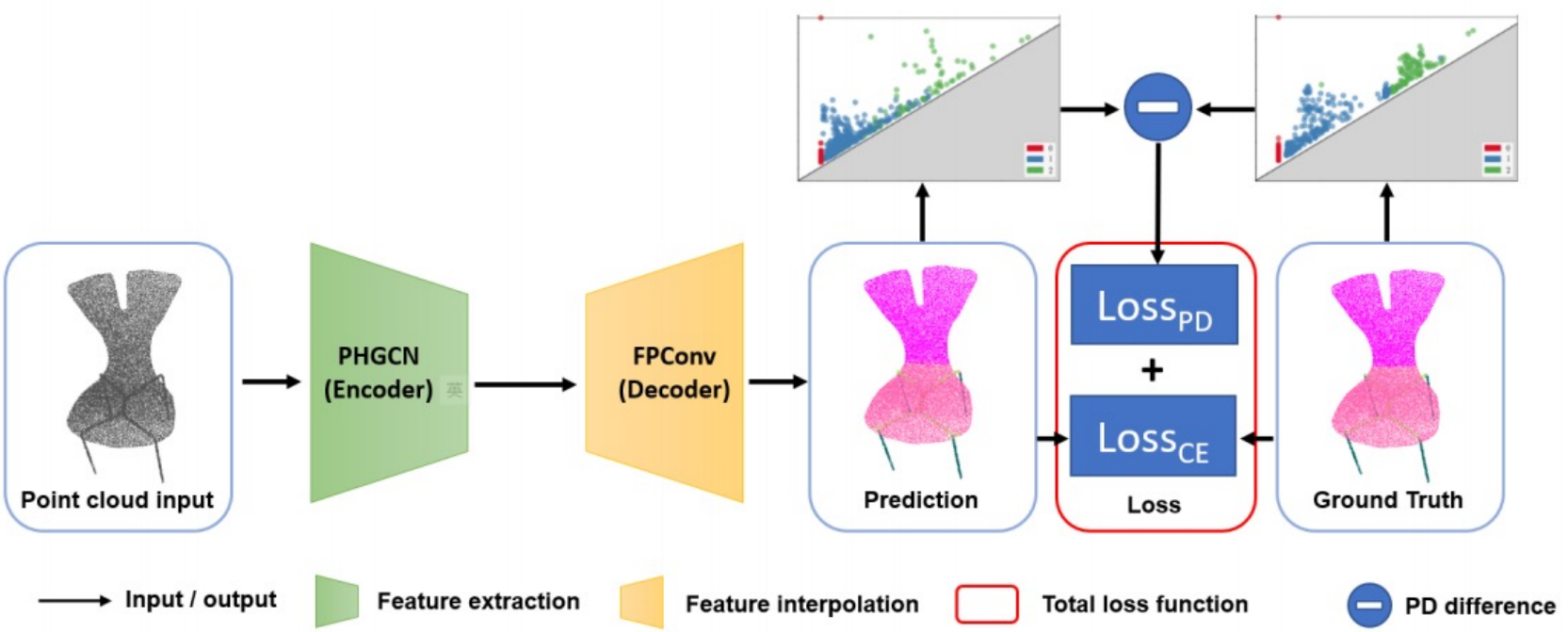
- 第三种方式是在损失函数中加入拓扑损失正则化项，约束其输出结果的拓扑性质
  - 在一些几何处理的任务中，一些拓扑特征（例如细小的孔洞）是关键要素，但是通常的深度学习的损失函数不能保证结果能够正确获取这些拓扑特征
  - 加入拓扑正则项则可以加上拓扑约束来实现这一点
- 常见的拓扑损失包括：持续度求和、两个PD的Wasserstein距离等

$$\mathcal{L}_{\text{total}} = \mathcal{L}_{\text{task}} + \lambda \cdot d_W(\text{PD}_{\text{pred}}, \text{PD}_{\text{gt}})$$



将拓扑融入损失函数的设计

- 例：PHGCN
- PHGCN将预测结果的PD和真实结果的PD之间的Wasserstein距离作为拓扑正则项加入损失函数里，用于增强点云分割的效果，特别是结果的拓扑正确性



Wong C C, Vong C M. Persistent homology based graph convolution network for fine-grained 3d shape segmentation[C]//Proceedings of the IEEE/CVF international conference on computer vision. 2021: 7098-7107.

05

拓扑理解与拓扑控制



主要内容

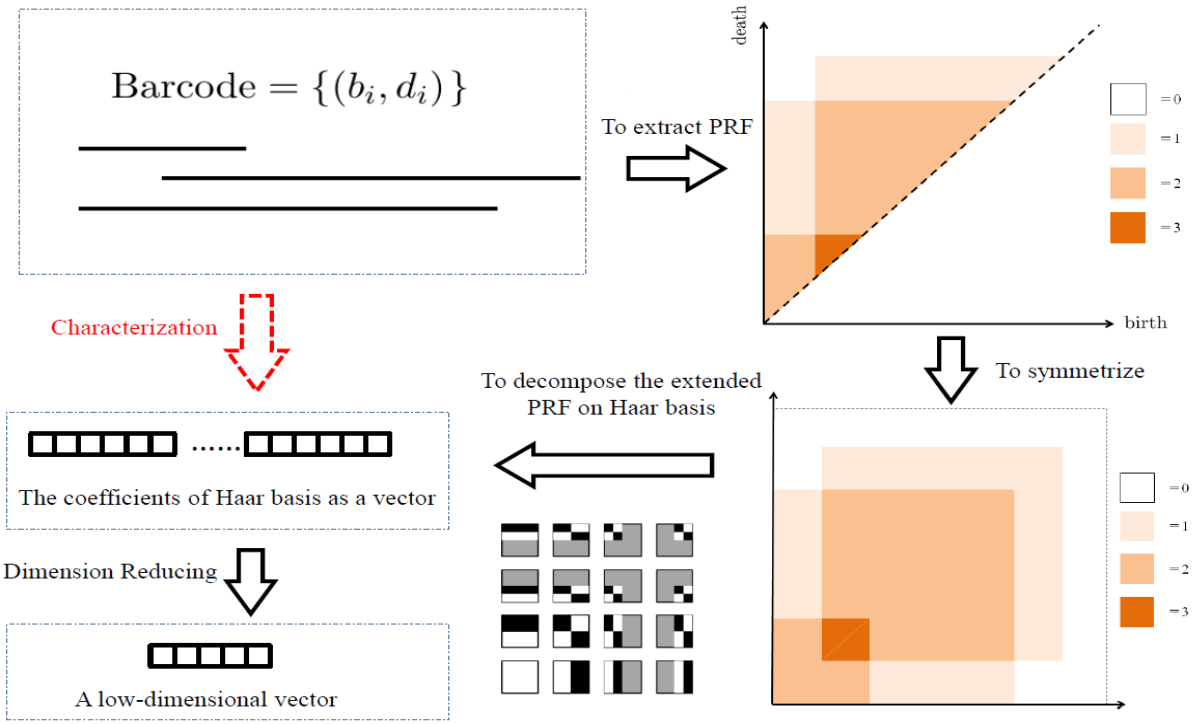
- 微结构检索
- 微结构设计
- 微结构三维打印
- 曲线曲面重建
- 视知觉计算模型
- 神经网络的可解释性
- 股票数据处理



计算拓扑是微结构处理的有力工具

Characterization of Persistence Barcode by Haar Basis

- 将Barcode转化为Extended PRF
- 将Extended PRF小波分解
- 小波分解系数降维，生成描述子



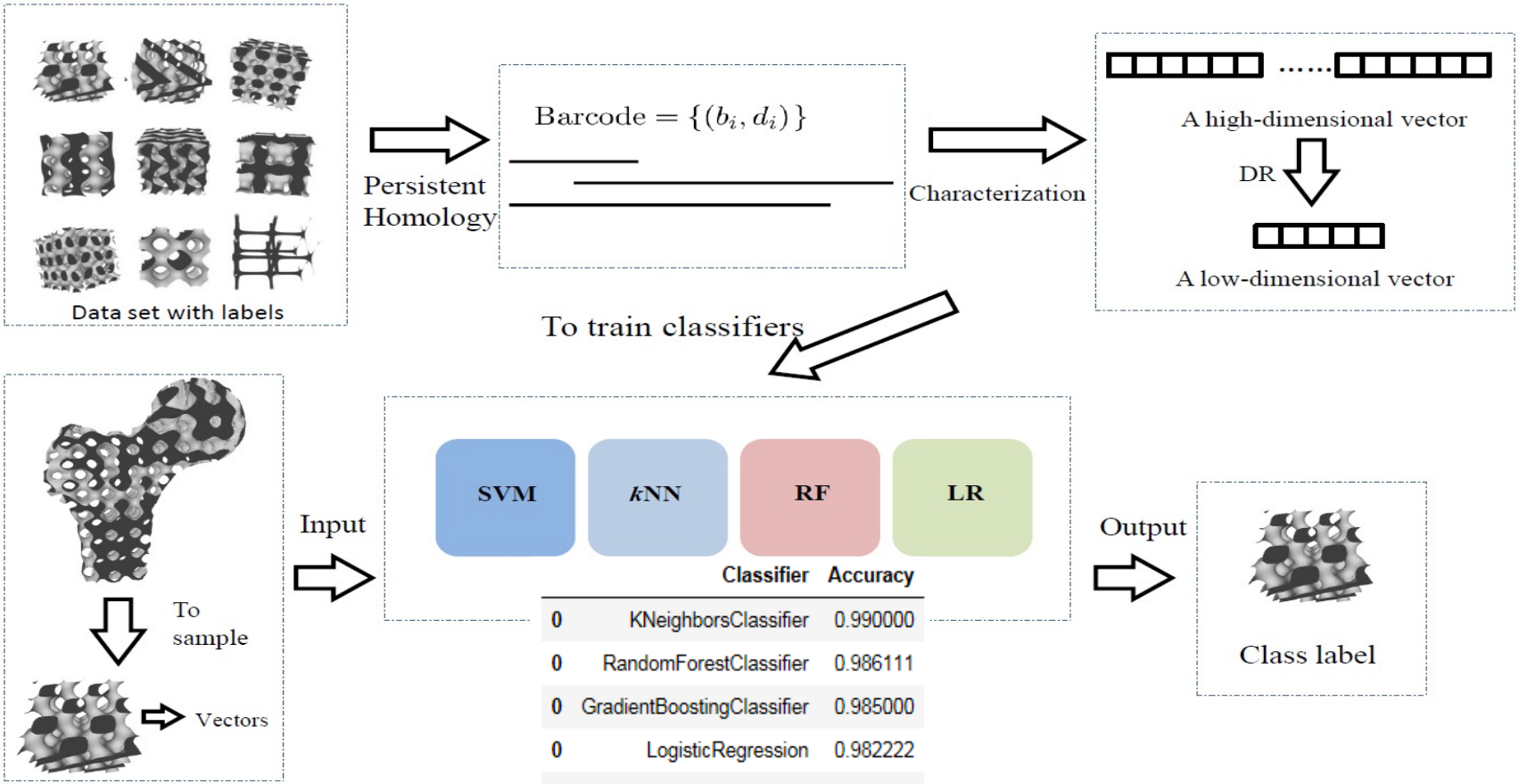
- Given Barcode$\{(b_i, d_i)|0 \leq b_i \leq d_i, i \in I, |I| < \infty\}$
- extended persistent rank function is defined as:

$$\tilde{r}_i(s,t) = \begin{cases} 1, & b_i \leq s, t \leq d_i \\ 0, & otherwise \end{cases}$$

Zhetong Dong, Chuanfeng Hu, Chi Zhou, Hongwei Lin, Vectorization of persistence barcode with applications in pattern classification of porous structures , Computers & Graphics, Volume 90, 2020,Pages 182-192

求是创新

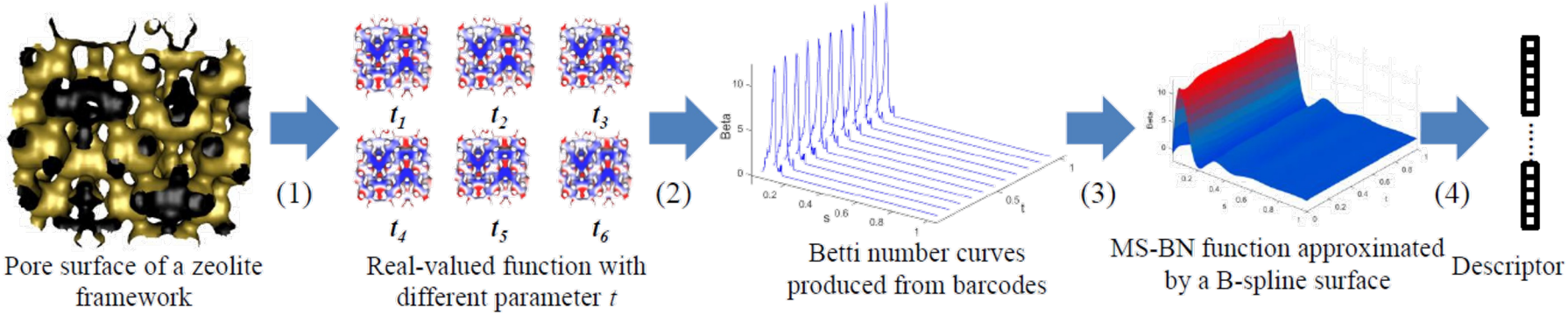
Characterization of Persistence Barcode by Haar Basis

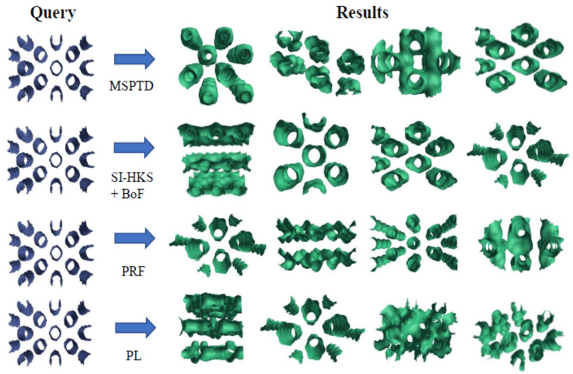


	Classifier	Accuracy
0	KNeighborsClassifier	0.990000
0	RandomForestClassifier	0.986111
0	GradientBoostingClassifier	0.985000
0	LogisticRegression	0.982222
0	LinearSVC	0.981667
0	SVC	0.990000

Multiscale Persistent Topological Descriptor

- 在多孔表面上定义热核函数，采样若干时刻，生成每一时刻对应温度场的Betti数曲线
- 用蒙皮算法，生成插值这些曲线的均匀B样条曲面
- B样条曲面的控制网格作为描述子





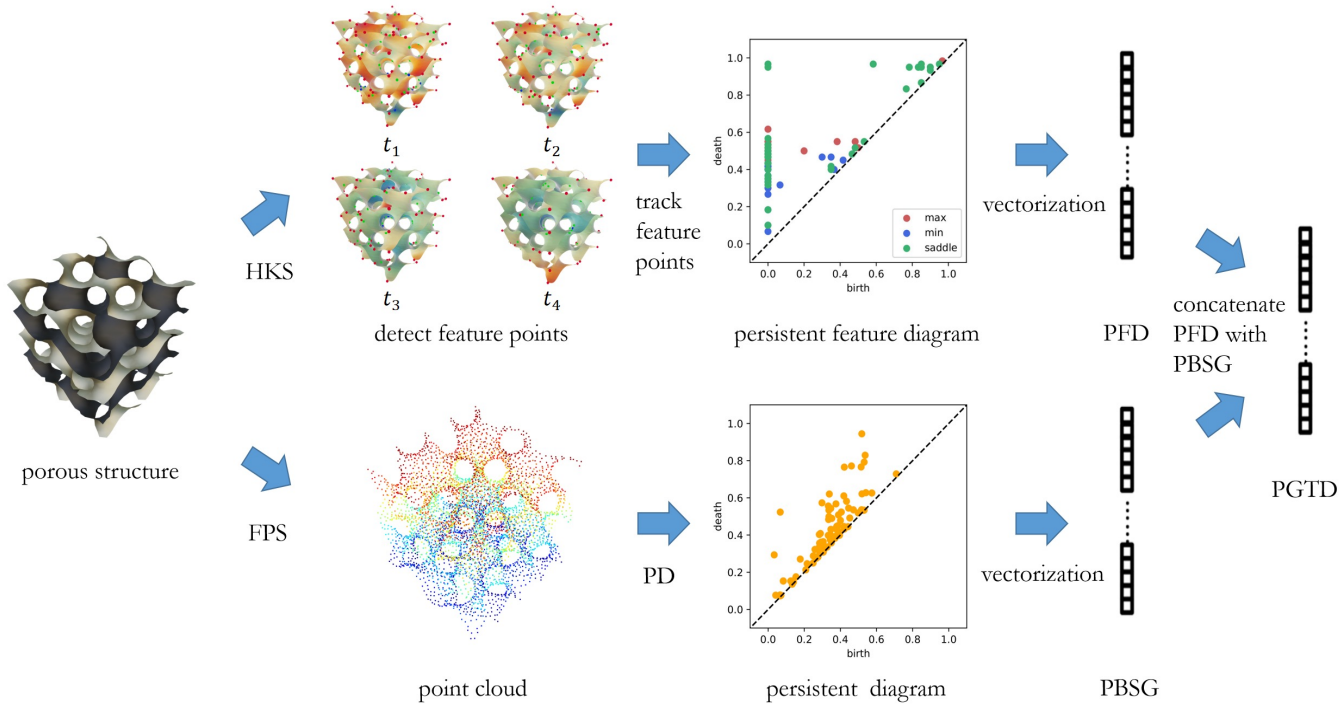
Methods/Accuracy (%)	NN	FT	ST	DCG
SI-HKS with histogram	90.1	73.3	91.6	92.7
SI-HKS with BoF	81.3	76.1	91.0	91.8
PL descriptor	68.1	59.3	75.0	83.4
PI descriptor	81.5	71.8	86.7	90.1
PRF descriptor	81.8	74.1	87.5	90.6
MSPTD (ours)	**94.9**	**78.4**	**94.5**	**95.5**

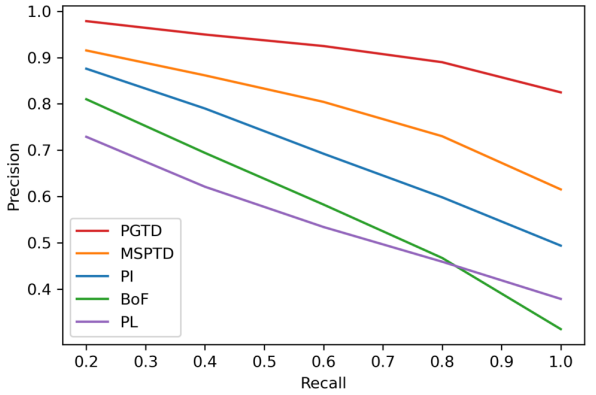
Zhetong Dong, Junyu Pu, Hongwei Lin. Multiscale Persistent Topological Descriptor for Porous Structure Retrieval. Computer Aided Geometric Design, 88: 102004 (2021) (SIAM-GD 2021)

求是创新

Persistent Geometry-Topology Descriptor：Pipeline

- 跟踪热核函数的特征点的出现和消失时刻，形成persistent feature diagram(PFD)，代表几何信息
- 持续图(PD)表示拓扑信息
- PD和PFD向量化后，形成Persistent Geometry-Topology Descriptor



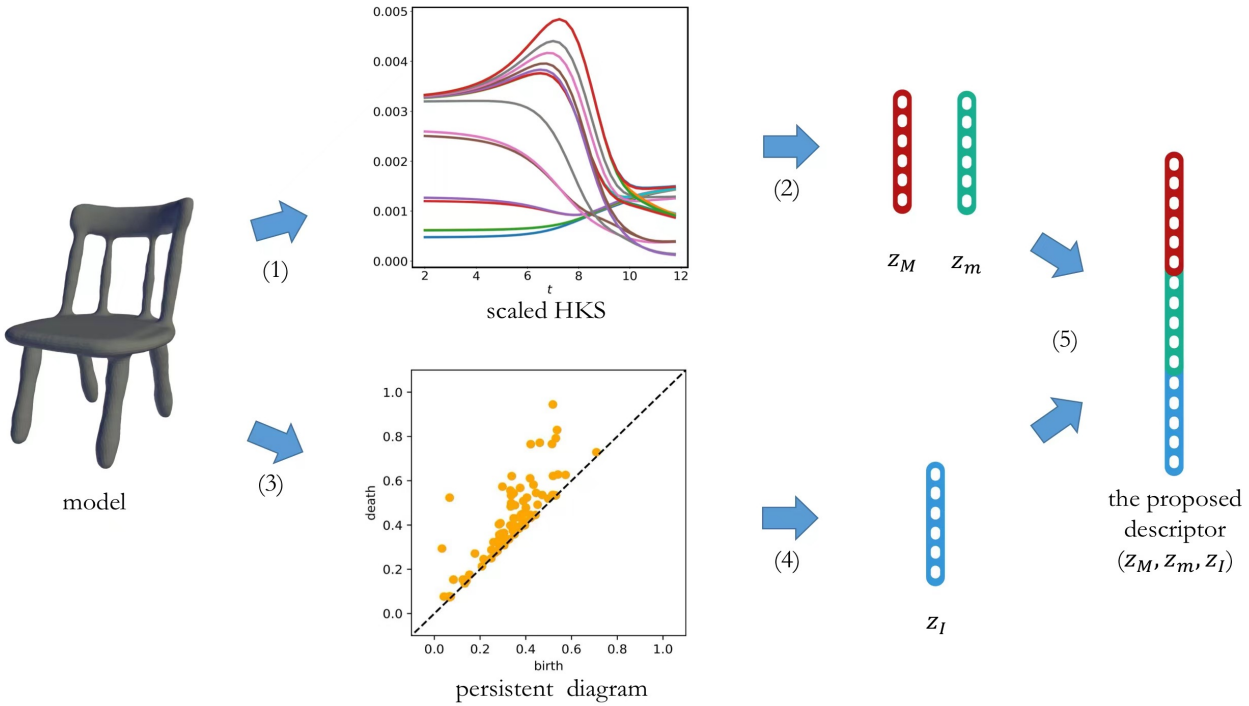


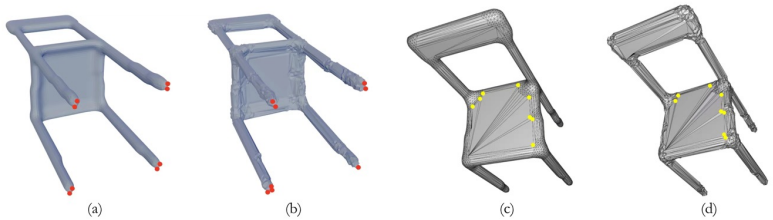
Methods/Accuracy (%)	NN	FT
MSPTD	86.8	71.0
HKS with BoF	75.1	50.8
PL descriptor	62.8	48.9
PI descriptor	81.6	61.8
PGTD (ours)	**96.3**	**86.5**

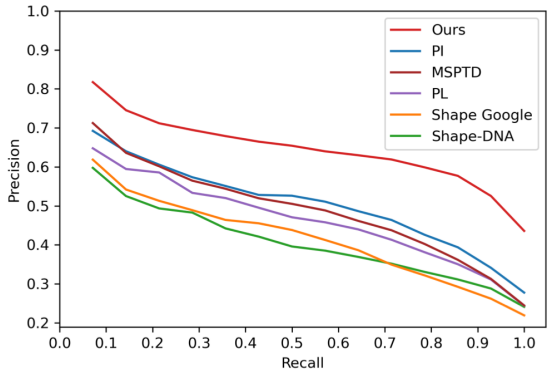
Peisheng Zhuo, Zitong He, Hongwei Lin. Persistent geometry-topology descriptor for porous structure retrieval based on heat kernel signature. Graphical Models, 133 (2024), 101219

Persistent Heat Kernel Signature Descriptor: pipeline

- 计算每个网格顶点的Heat Kernel Signature曲线，以及曲线上的最大值与最小值
- 将极大值排序，极小值排序，与PD的向量化合并成描述子







Methods/Accuracy (%)	NN	FT	ST
Shape-DNA (Reuter et al., 2006)	46.49	37.66	53.01
Shape Google (Bronstein et al., 2011)	52.63	37.34	52.57
PI (Adams et al., 2017)	61.40	44.67	62.47
PL (Bubenik, 2015)	57.89	40.85	58.02
MSPTD (Dong et al., 2021)	60.53	43.92	62.66
Our descriptor	**78.07**	**59.09**	**73.56**

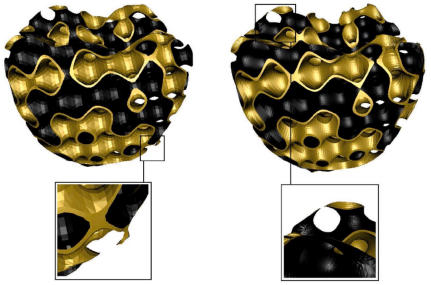
Zitong He, Peisheng Zhuo, Hongwei Lin, Junfei Dai. 3D shape descriptor design based on HKS and persistent homology with stability analysis. Computer Aided Geometric Design, 111 (2024) 102326

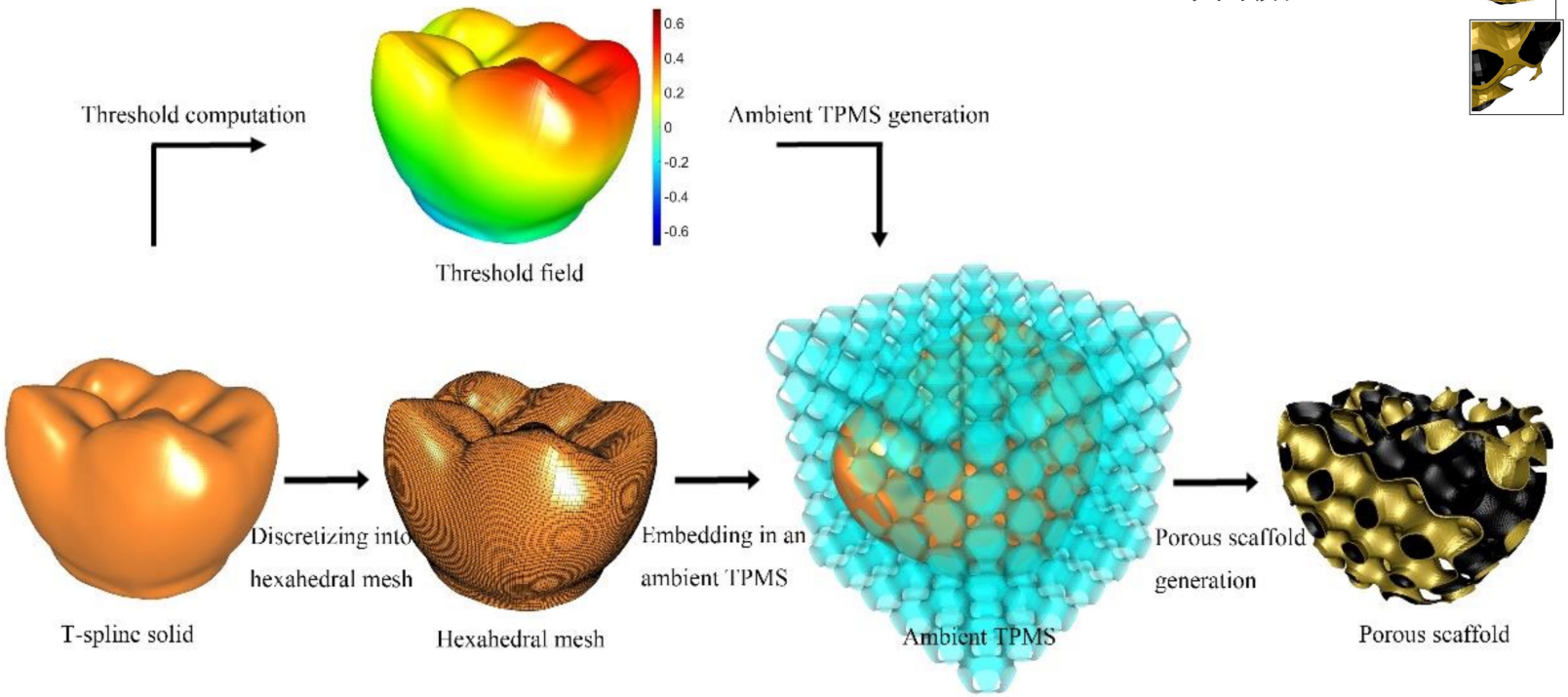
求是创新

多孔结构设计中的拓扑理解与拓扑控制

- 传统自由形状多孔结构生成方法

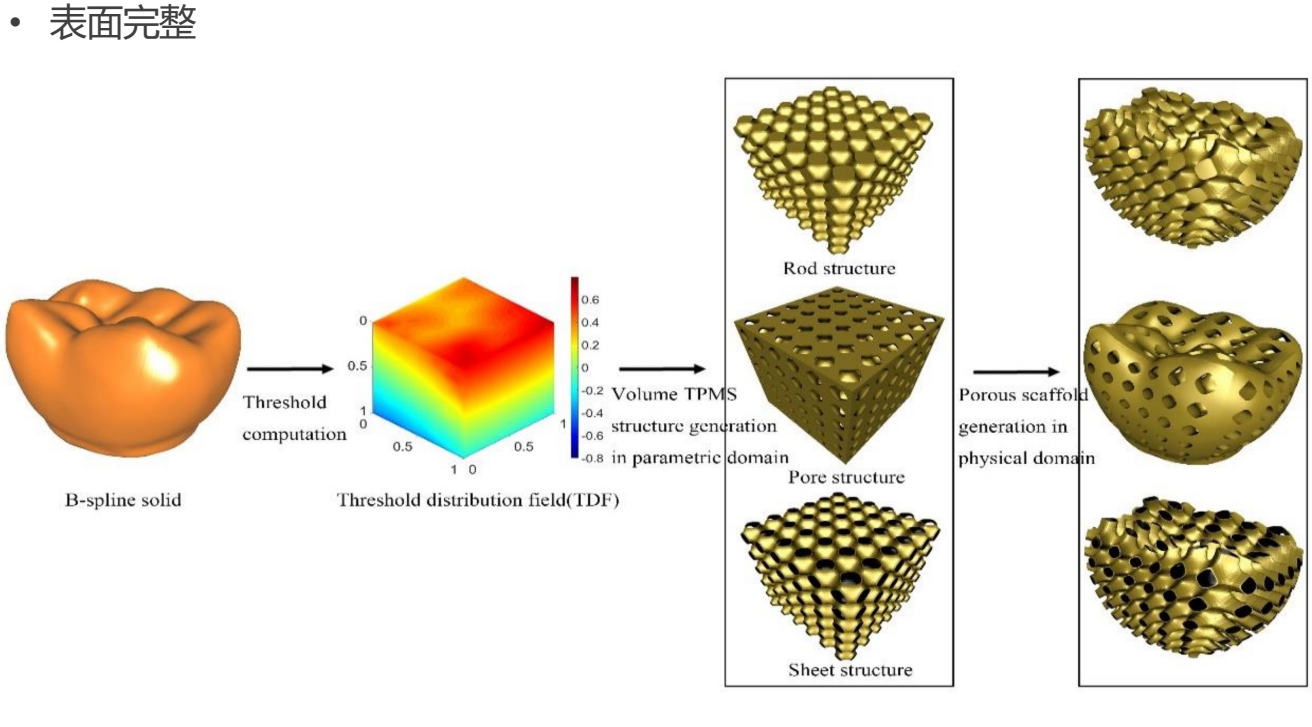
- 存储量大
- 表面破裂

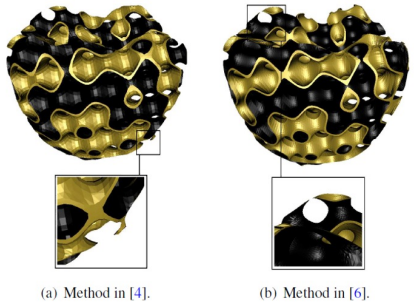


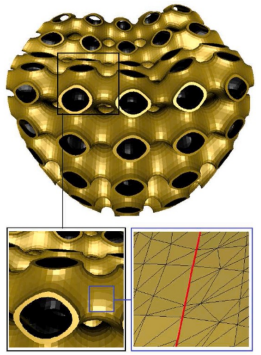


多孔结构设计中的拓扑理解与拓扑控制

- 新方法：参数域中的隐式B样条与自由外形的三变量B样条体复合
- 节省存储空间99%以上
- 表面完整



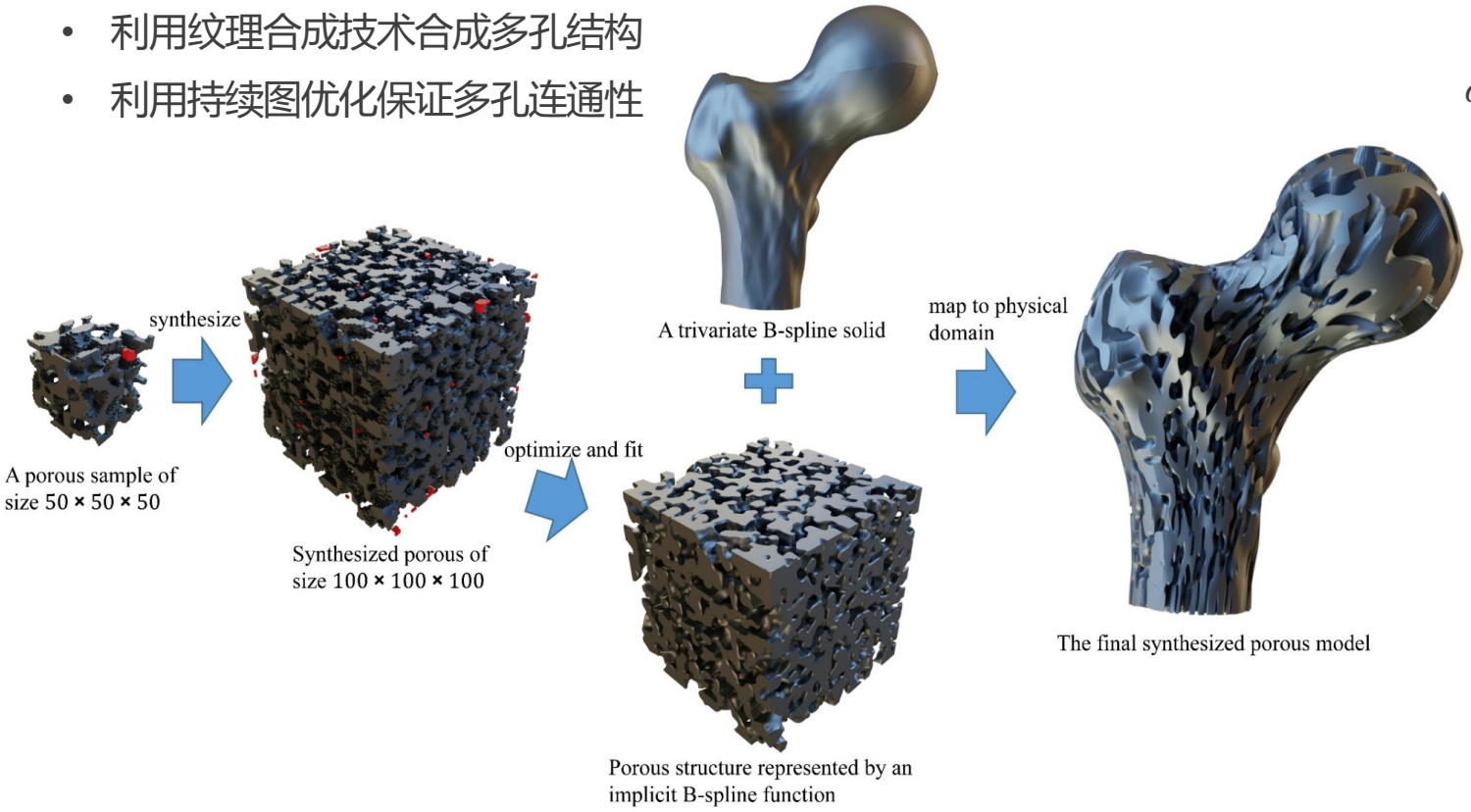


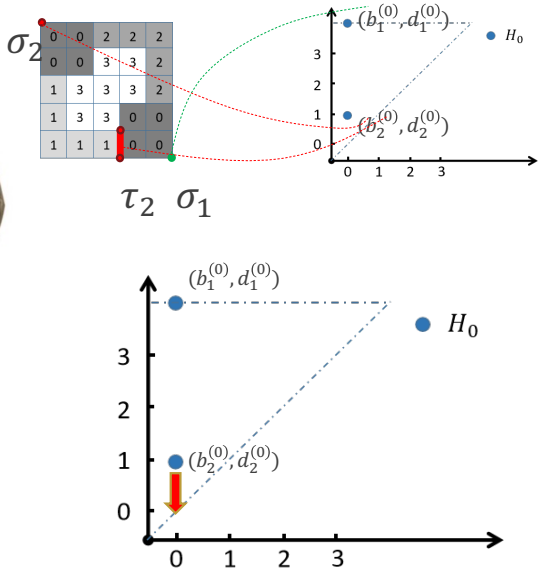


Chuanfeng Hu, Hongwei Lin. Heterogeneous porous scaffold generation using trivariate B-spline solids and triply periodic minimal surfaces. Graphical Model. 115: 101105, 2021

Connectivity-guaranteed porous synthesis

- 利用纹理合成技术合成多孔结构
- 利用持续图优化保证多孔连通性



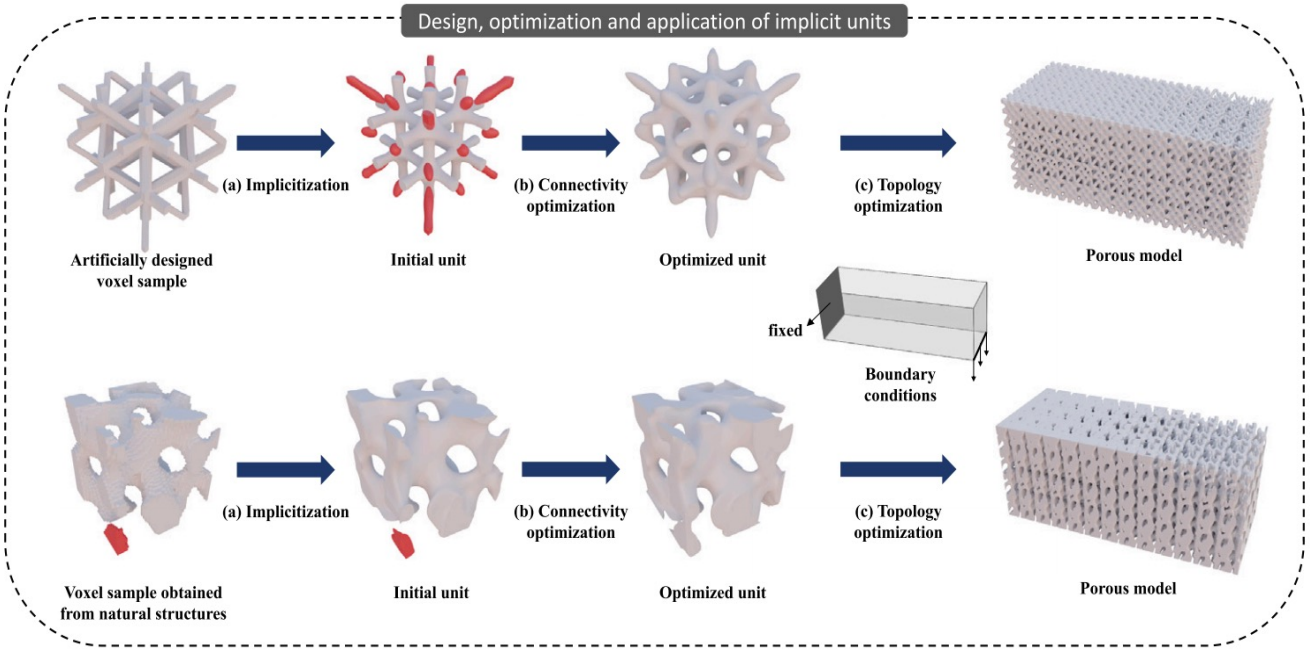


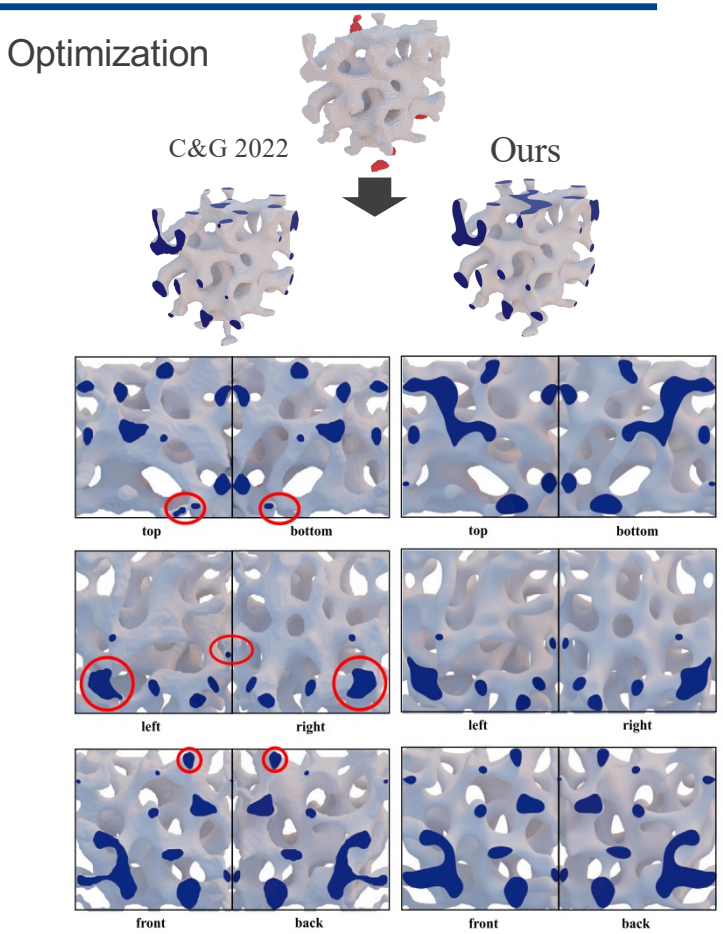
- DTM based filtration
- PD optimization

Depeng Gao, Jinhao Chen, Zhetong Dong, Hongwei Lin. Connectivity-guaranteed porous synthesis in free form model by persistent homology. Computers & Graphics, 106: 33-44 (2022)

Periodic Implicit Representation: Flowchart

- 采用周期隐式B样条表示多孔结构单元
- 单元之间可以自然拼接
- 丰富了多孔结构的表示形式和设计手段



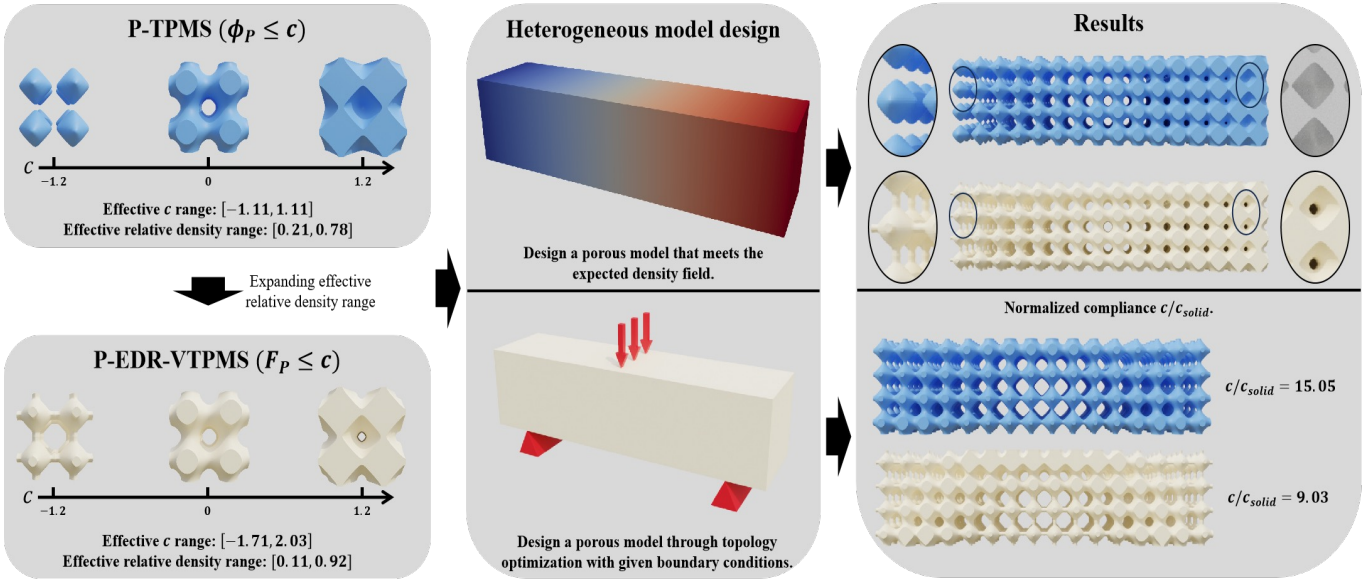


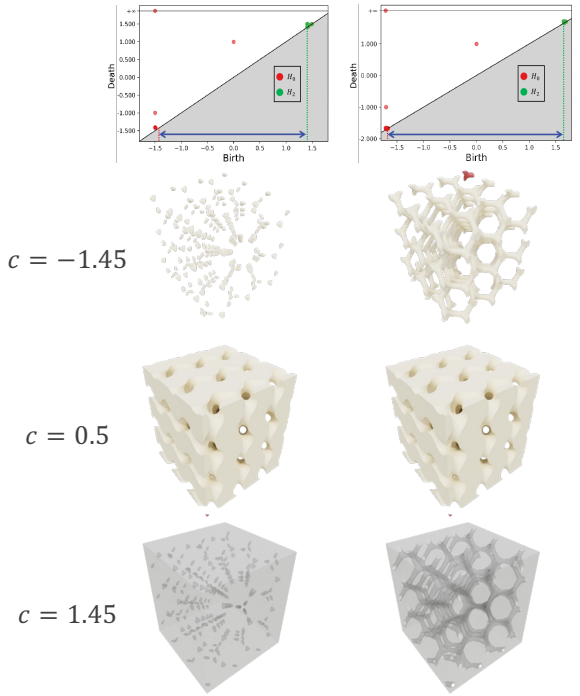
Depeng Gao, Yang Gao, Hongwei Lin. Periodic implicit representation, design and optimization of porous structure using periodic B-splines. Computer-Aided Design, 171 (2024) 103703

求是创新

EDR-VTPMS: Flowchart

- TPMS公式为$\Phi(x) < C$， $\Phi(x)$为三角函数，可调参数少， C有有效区间
- 用B样条函数代替$\Phi(x)$，利用持续图优化，扩展C的有效区间





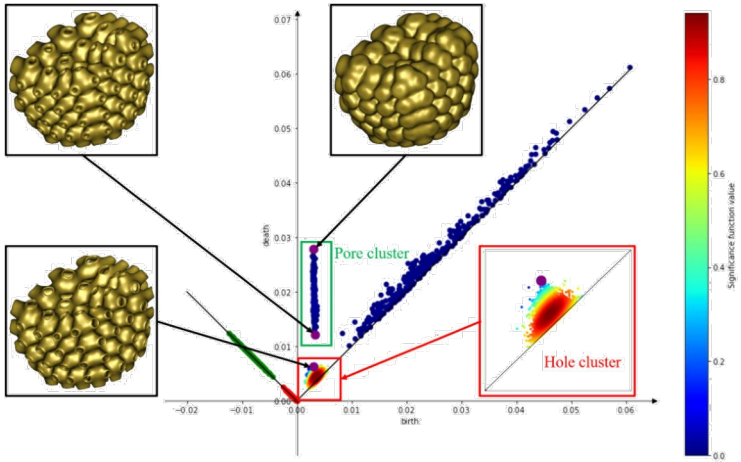
EDR-VTPMS：Effective relative density range-Variant TPMA

Depeng Gao, Yuanzhi Zhang, Hongwei Lin. Persistent Homology-Driven Optimization of Effective Relative Density Range for Triply Periodic Minimal Surface. Submitted

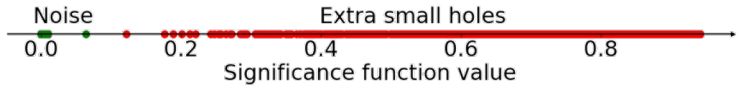
求是创新

Reasonable thickness determination for sheet structure

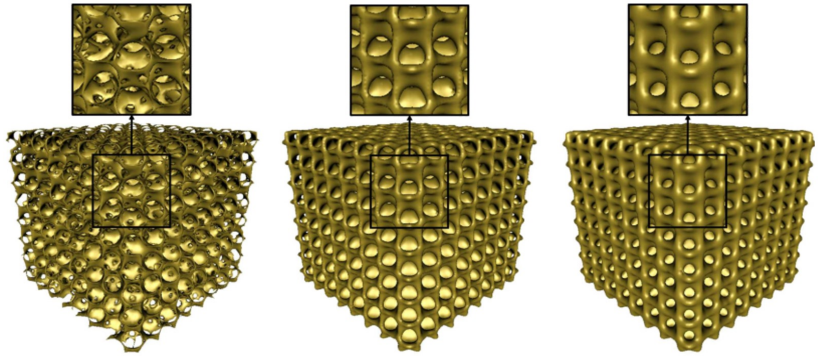
- TPMS是一张曲面，要打印TPMS，必须使它有厚度
- 厚度过小，表面上会有额外小洞；厚度过大，多孔会封闭
- 通过对持续图聚类，确定合理的厚度范围



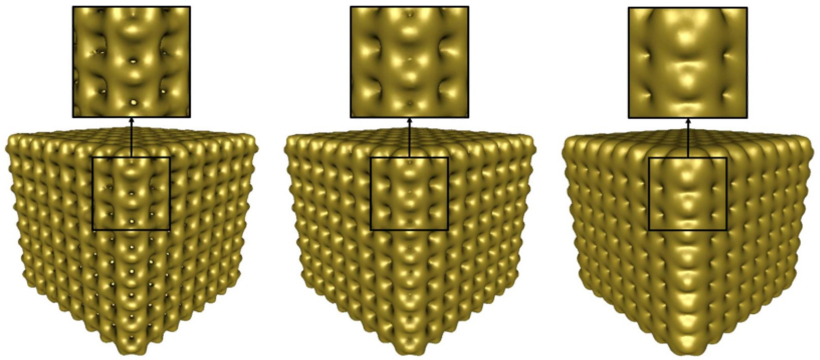
(a) 1-PD of the scalar field $|f(x, y, z)|$.



(b) Significance function values of the points in the short persistence cluster.



(b) $c = 6.00 \times 10^{-3}$. (c) $c = c_{min} = 9.29 \times 10^{-3}$. (d) $c = 1.50 \times 10^{-2}$.

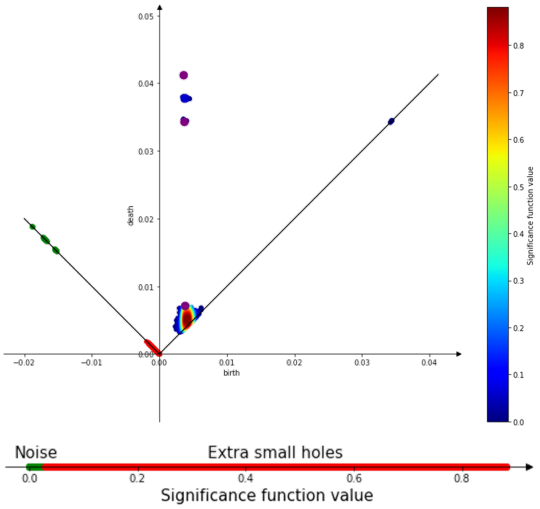


(e) $c = c_{max,1} = 2.26 \times 10^{-2}$. (f) $c = c_{max,2} = 2.58 \times 10^{-2}$. (g) $c = 4.00 \times 10^{-2}$.

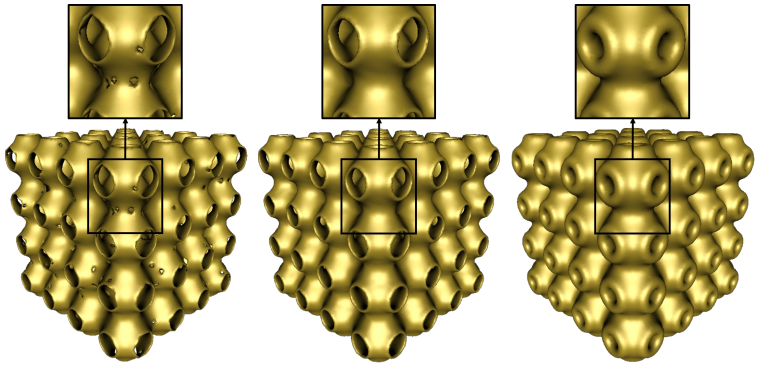
Jiacong Yan, Hongwei Lin. Reasonable thickness determination for implicit porous sheet structure using persistent homology. Computers & Graphics, 115, 236-245, 2023 (CAD/Graphics 2023)

Reasonable thickness determination for porous sheet structure

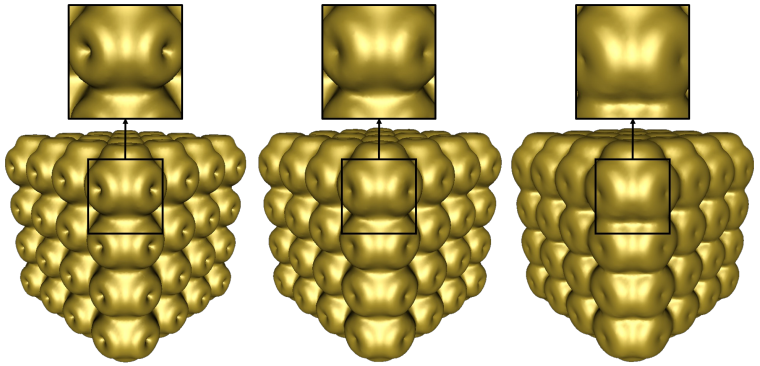
Experimental Results:



(1) 1-PD of scalar field $|f(x,y,z)|$.



(2) $c = 6.00 \times 10^{-3}$. (3) $c = c_{min} = 7.20 \times 10^{-3}$. (4) $c = 2.00 \times 10^{-2}$.



(5) $c = c_{max,1} = 3.43 \times 10^{-2}$. (6) $c = c_{max,2} = 4.13 \times 10^{-2}$. (7) $c = 6.00 \times 10^{-2}$.

The variation of the number of pores and holes in these sheet structures

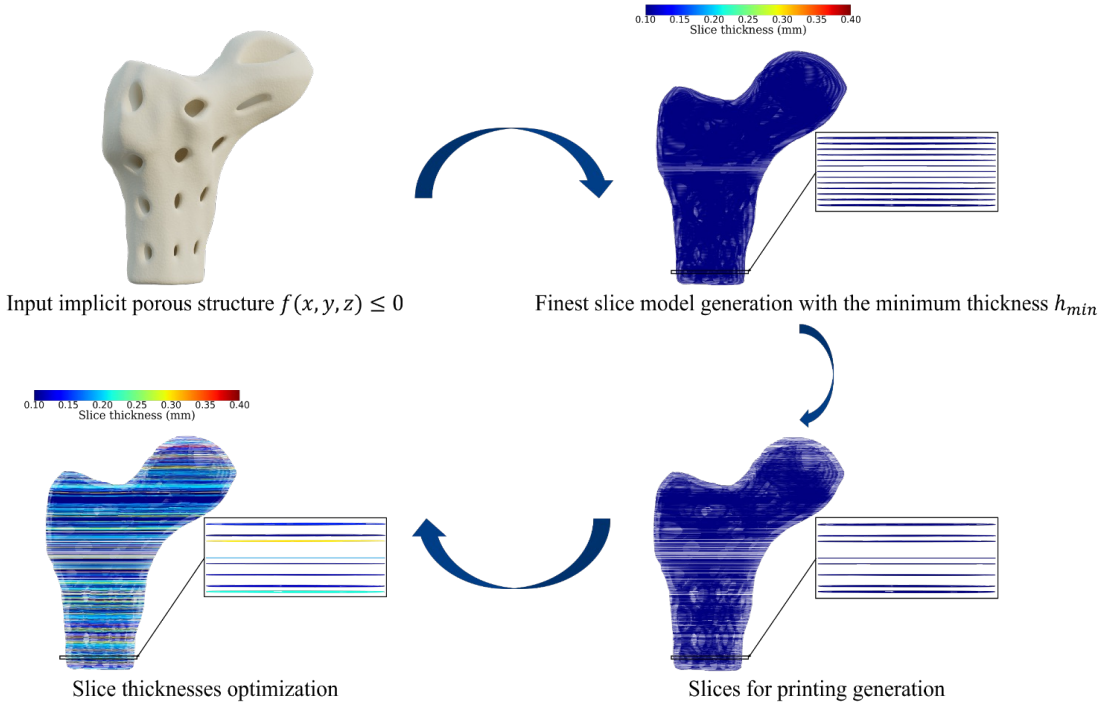
Parameter c	Topological measurement $\bar{\beta}$
$c < c_{min}$	5.661
$c = c_{min}$	**1.000**
$c_{min} < c < c_{max,1}$	**1.000**
$c = c_{max,1}$	0.998
$c = c_{max,2}$	0.000
$c > c_{max,2}$	0.000

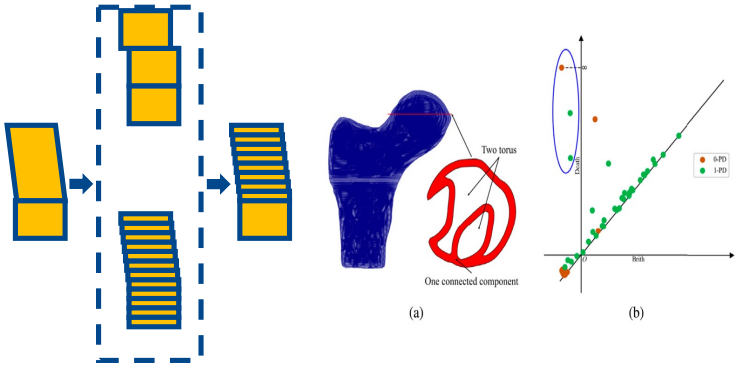
Jiacong Yan, Hongwei Lin. Reasonable thickness determination for implicit porous sheet structure using persistent homology. Computers & Graphics, 115, 236-245, 2023

求是創新

Adaptive slicing with topology guarantee

- 多孔结构的2维切片也具有复杂的拓扑结构
- 通过分析比较相邻切片的持续图，找到发生拓扑突变的关键切片位置
- 保证打印模型与设计模型的拓扑一致性





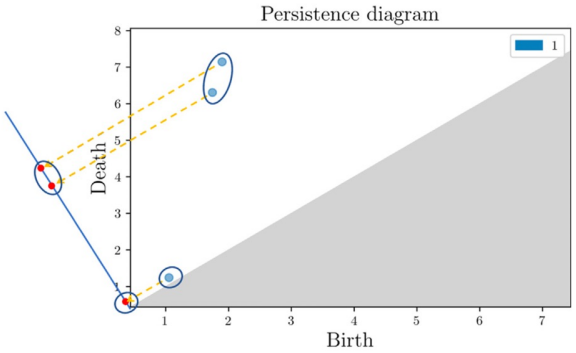
Model	Method	Slice number	Topology detection error	Cusp height sum (mm)	Volume deviation
Balljoint	**Our method**	502	**0**	**44.34**	**0.31%**
	Cusp height based method	502	192	47.52	0.39%
	Boolean operations based method	502	197	46.34	0.40%
Moai	**Our method**	580	**0**	**35.56**	**0.40%**
	Cusp height based method	580	192	37.45	0.41%
	Boolean operations based method	580	148	37.02	0.41%
Tooth	**Our method**	652	**0**	**54.63**	**0.58%**
	Cusp height based method	652	274	56.84	0.62%
	Boolean operations based method	652	233	55.93	0.62%
Venus	**Our method**	569	**0**	**51.31**	**0.55%**
	Cusp height based method	569	227	54.23	0.60%
	Boolean operations based method	569	264	53.20	0.61%

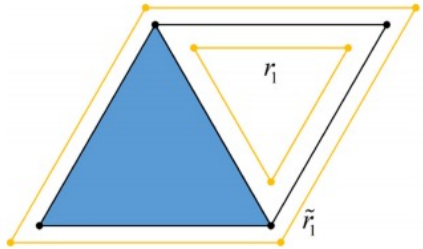
Jiacong Yan, Hongwei Lin. Adaptive Slicing of Implicit Porous Structure with Topology Guarantee. Computer-Aided Design, 162, 103557, 2023 (SPM 2023)

求是创新

Reconstruction of close curves by persistent homology

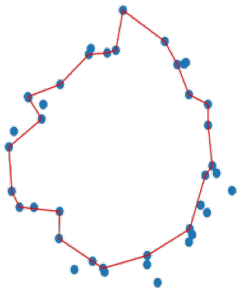
- 计算点云的PD，并对PD上的多重点集分类
- 计算显著点对应的持续1-闭链代表元，并作为初始曲线的控制多边形
- 利用LSPIA，生成重建曲线



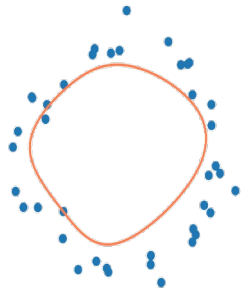




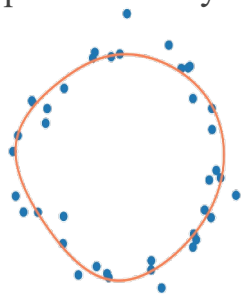
(a) Input point cloud.



(b) Extracted persistent 1-cycle.



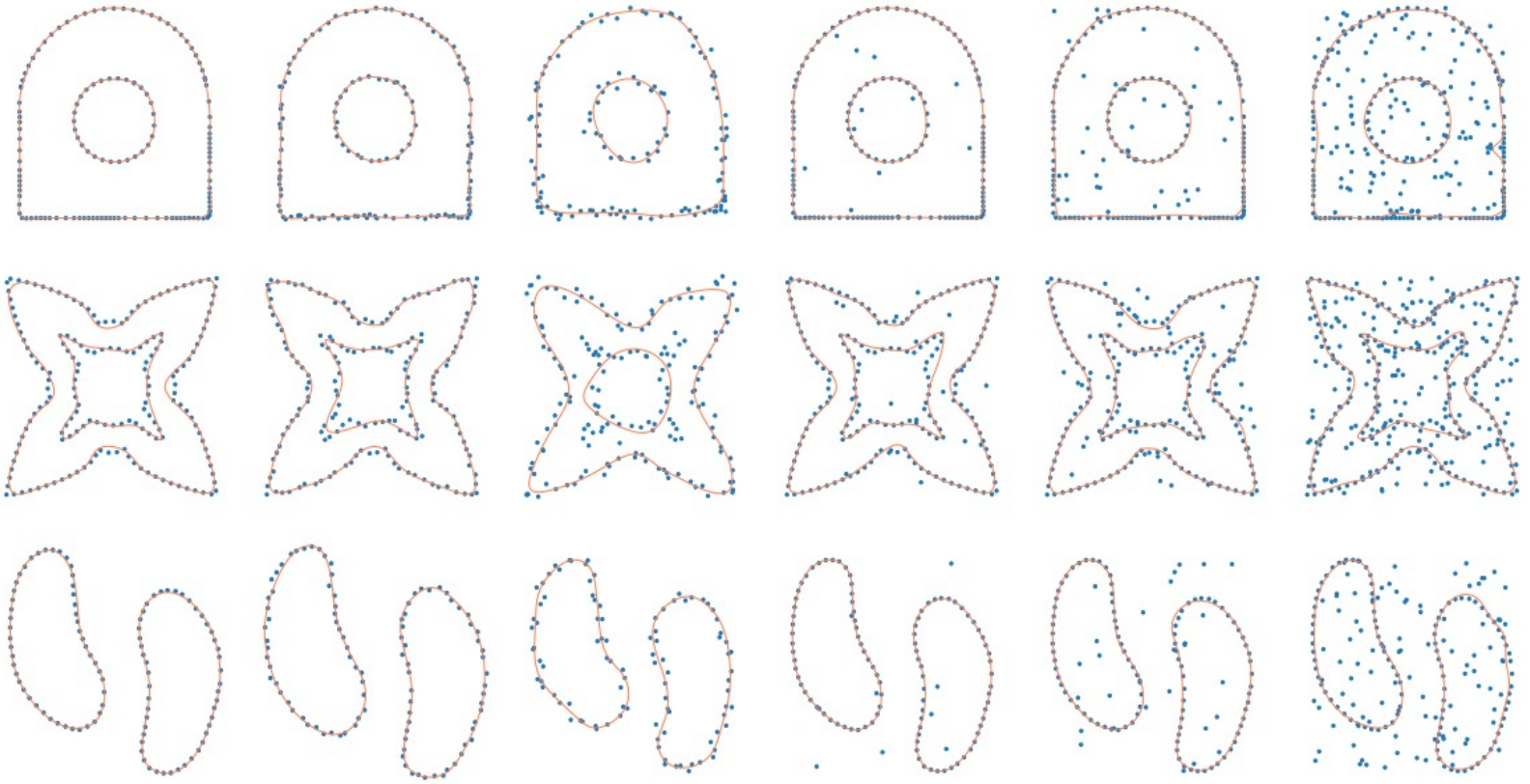
(c) Initial parametric B-spline curve.



(d) Reconstructed parametric B-spline curve.

Yaqi He, Jiacong Yan, Hongwei Lin. Robust reconstruction of closed parametric curves by topological understanding with persistent homology[J]. Computer-Aided Design, 2023, 163: 103611.

2D Closed Curve Reconstruction (Multiple loops)



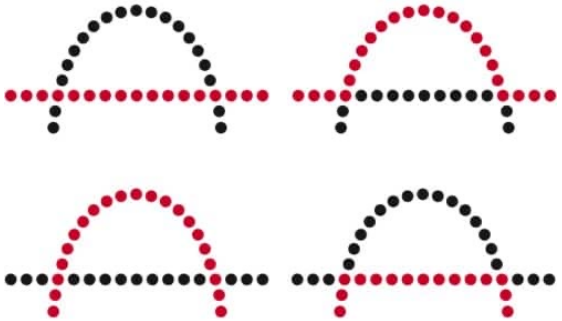
(a) Original points. (b) Points with noise (γ = 0.01). (c) Points with noise (γ = 0.03). (d) Points with 10% added outliers. (e) Points with 30% added outliers. (f) Points with 100% added outliers.

Yaqi He, Jiacong Yan, Hongwei Lin. Robust reconstruction of closed parametric curves by topological understanding with persistent homology[J]. Computer-Aided Design, 2023, 163: 103611.

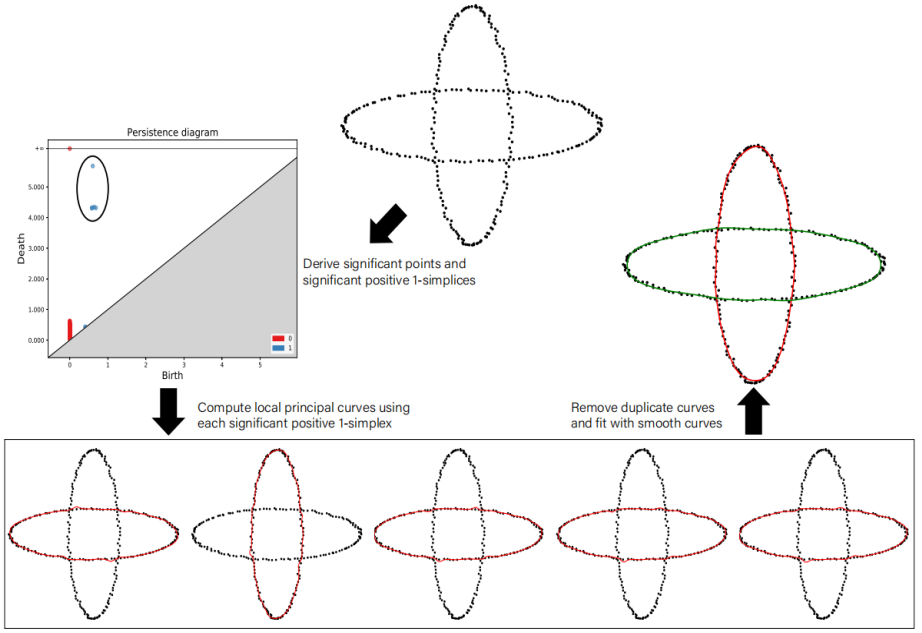
求是創新

Human perception faithful curve reconstruction

- 问题：重建符合人类视觉感知的曲线，解决相交曲线的重建问题
- 方法：
  - 计算点云的PD，并对PD中的点分类（显著点和噪声点）
  - 对每一个显著点，利用local principal curve重建符合格式塔good continuation原则的曲线
  - 删除重复重建的曲线

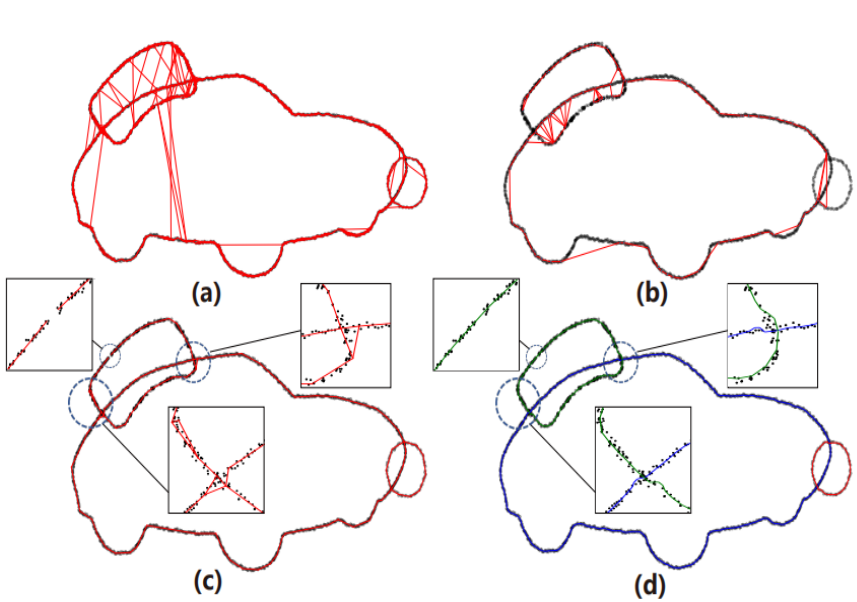


Gestalt good continuation principle

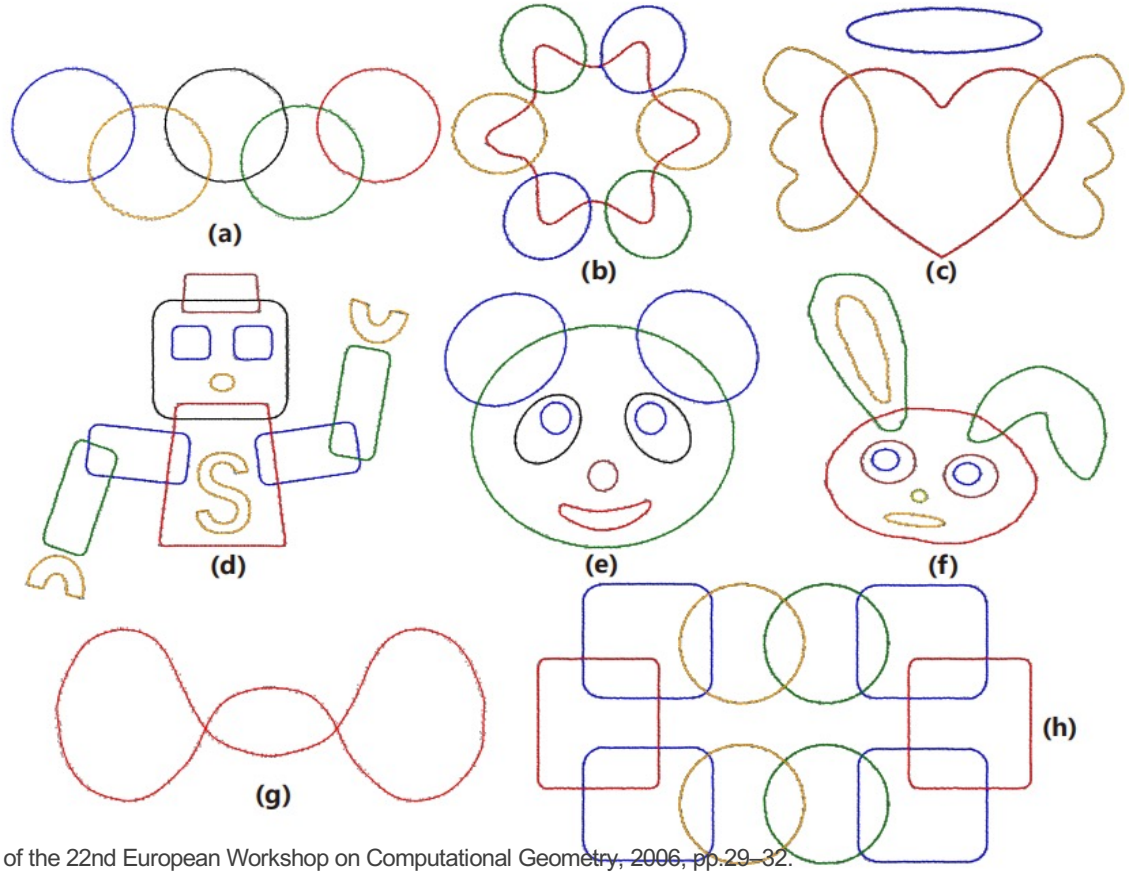


Yu Chen, Hongwei Lin, Yifan Xing. Human perception faithful curve reconstruction based on persistent homology and principal curve. Graphical Models, 139 (2025): 101267.

Results and Comparisons

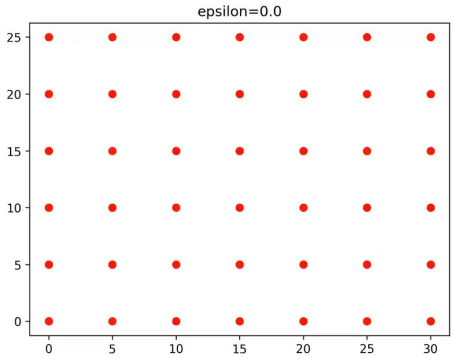


Comparisons

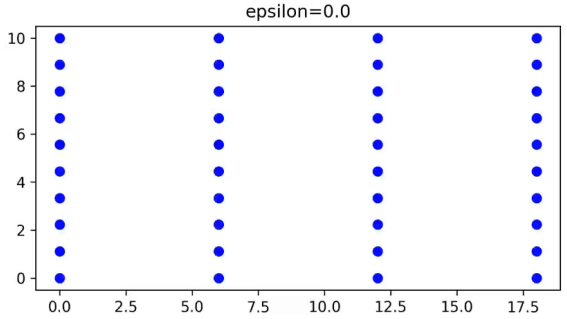


[a] T. Lenz, How to sample and reconstruct curves with unusual features, in: EWCG: Proc. of the 22nd European Workshop on Computational Geometry, 2006, pp.29–32.
[b] A.D. Parakkat, S. Methirumangalath, R. Muthuganapathy, Peeling the longest: A simple generalized curve reconstruction algorithm, Comput. Graph. (2018).
[c] F. De Goes, D. Cohen-Steiner, P. Alliez, M. Desbrun, An optimal transport approach to robust reconstruction and simplification of 2d shapes, in: Computer Graphics Forum, vol. 30, (5) Wiley Online Library, 2011, pp. 1593–1602.

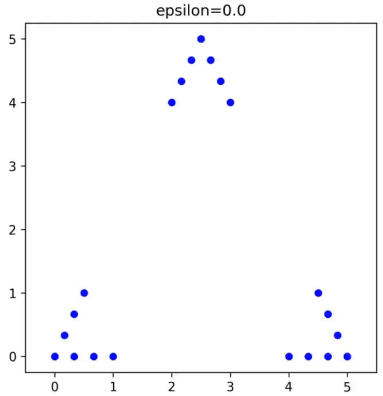
The Gestalt Computational Model



相似原则

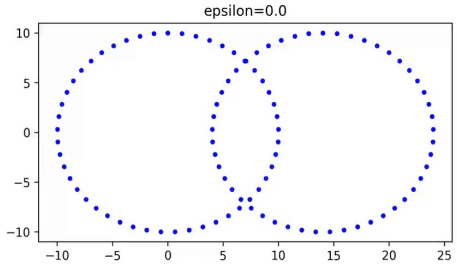


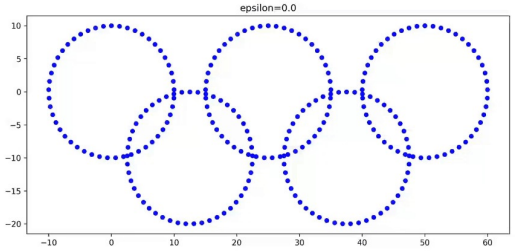
邻近原则



闭合原则

良好连续原则





简单原则

发展了格式塔原则的计算模型

Yu Chen, Hongwei Lin, Jiacong Yan. The Gestalt computational model by persistent homology. Vision Research, 237: 108697, 2025

More Results

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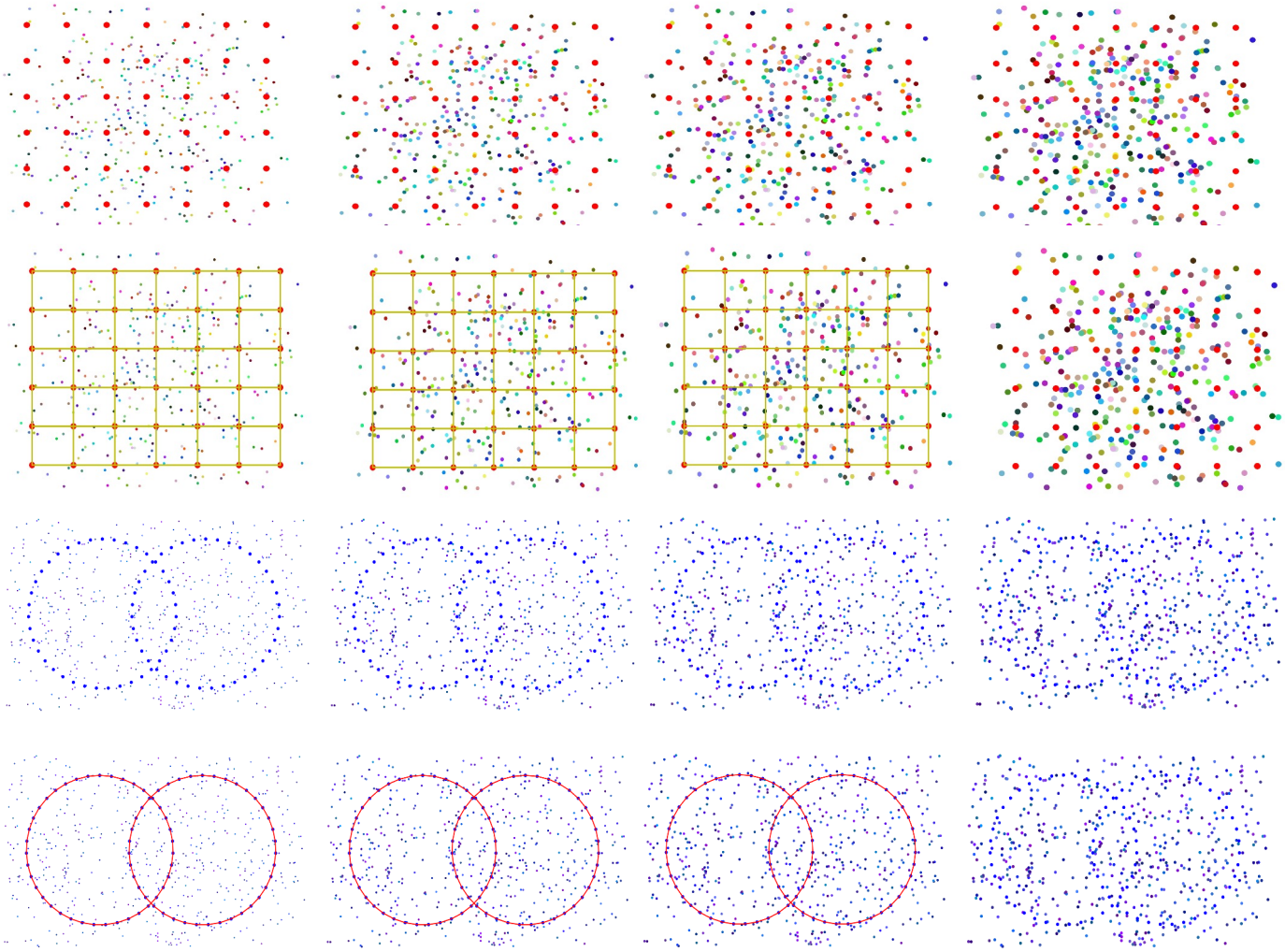
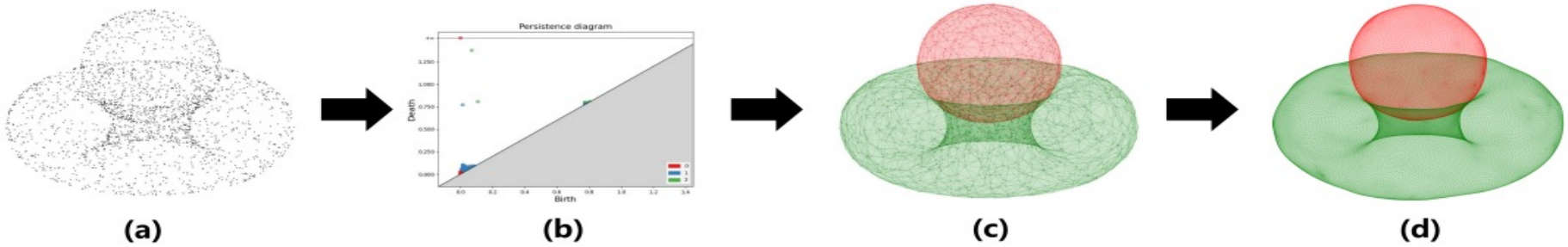
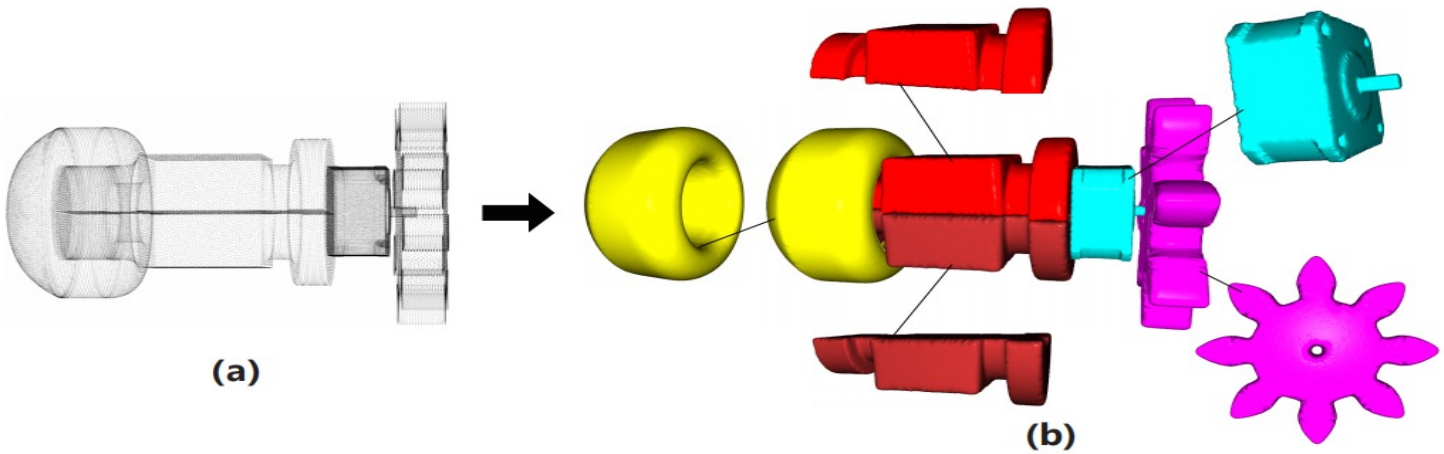
鲁棒性：



Robust Model Reconstruction

- 传统重建方法一次只能重建一张曲面
- 借助持续同调带来的拓扑理解能力，发展了一次重建多张曲面的模型重建技术





Yu Chen, Hongwei Lin. Robust Model Reconstruction Based on the Topological Understanding of Point Clouds Using Persistent Homology. Computer-Aided Design, 189: 103934, 2025

求是创新

Comparison

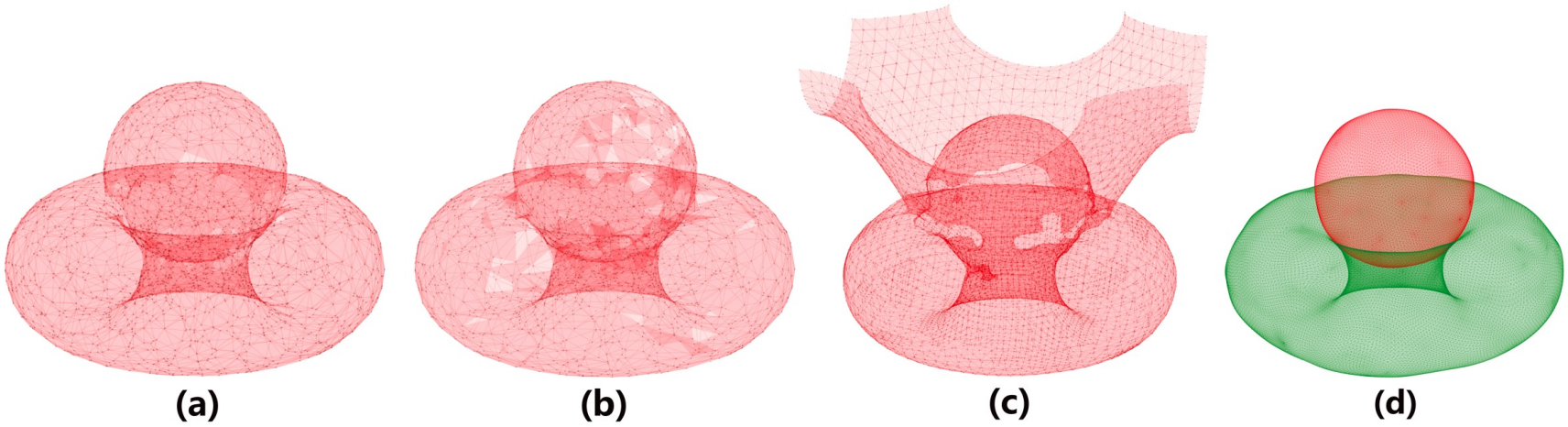
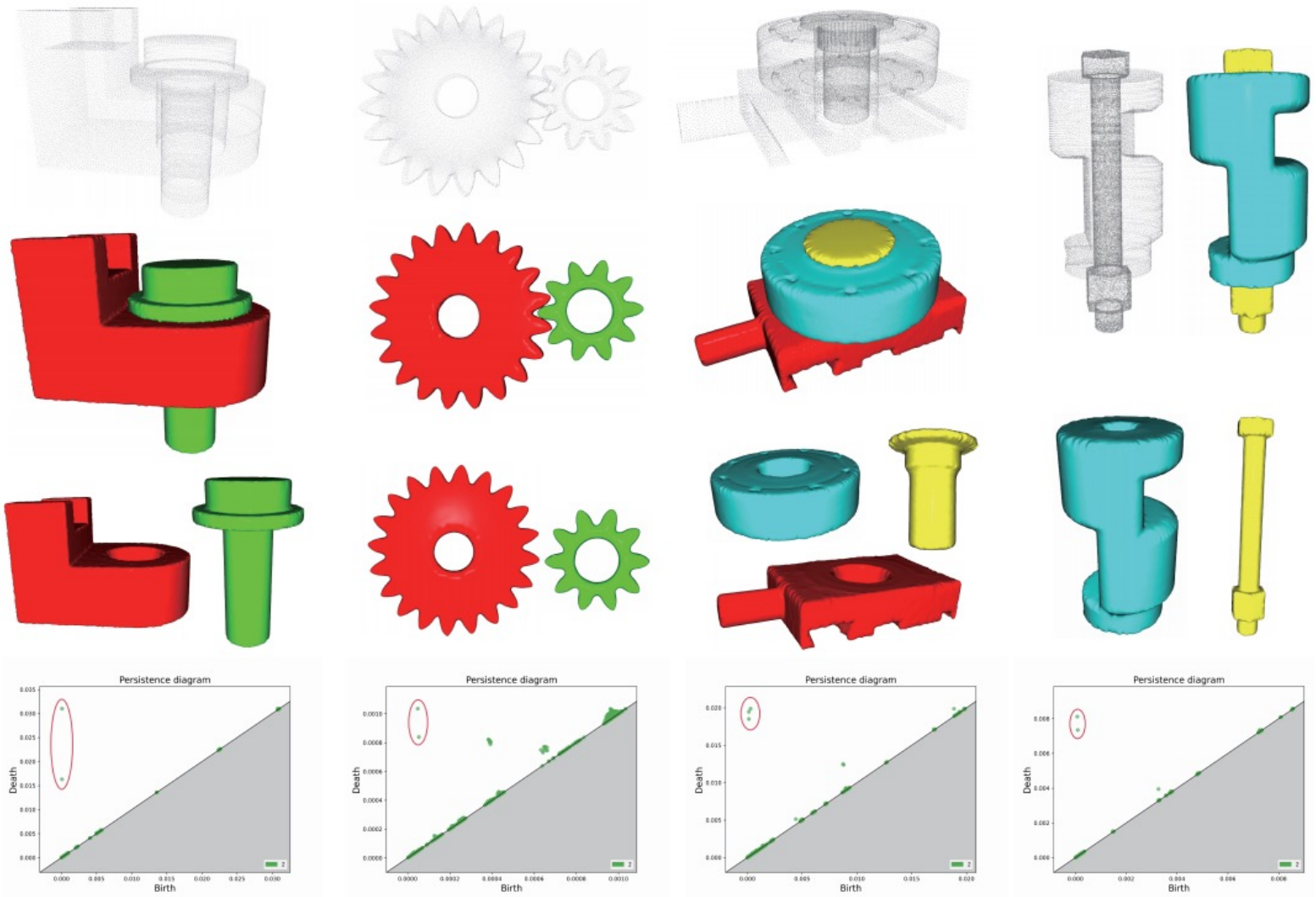


Figure 12: The comparison results. (a) Result of the greedy Delaunay method. (b) Result of the ball-pivoting method. (c) Result of the screened Poisson surface reconstruction method. (d) Result of our method.

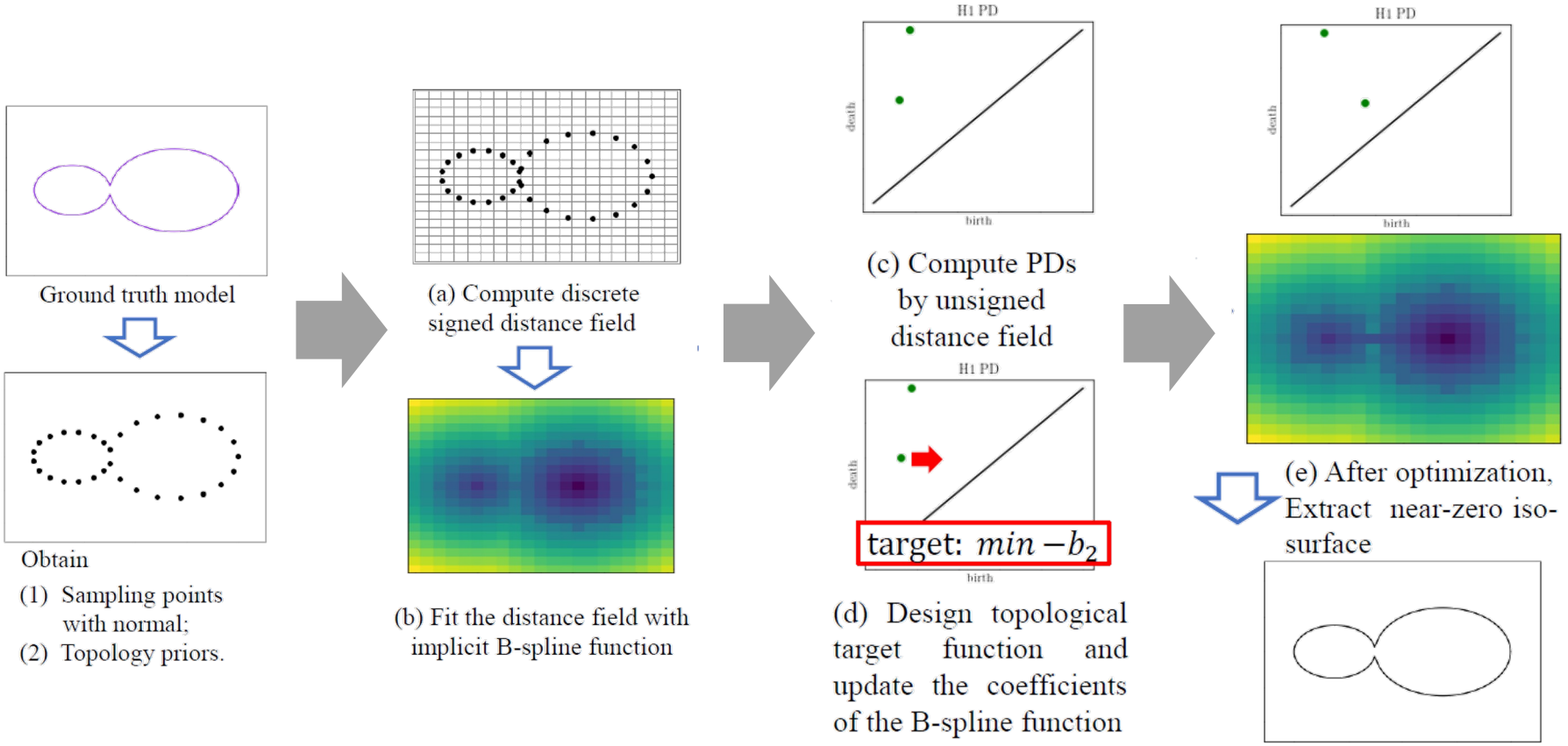
Comparisons

[a] D. Cohen-Steiner, F. Da, A greedy delaunay-based surface reconstruction algorithm, The visual computer 20 (2004) 4–16.
[b] F. Bernardini, J. Mittleman, H. Rushmeier, C. Silva, G. Taubin, The ballpivoting algorithm for surface reconstruction, IEEE transactions on visualization and computer graphics 5 (4) (1999) 349–359.
[c] M. Kazhdan, H. Hoppe, Screened poisson surface reconstruction, ACM Transactions on Graphics (ToG) 32 (3) (2013) 1–13

Results of Reconstructing CAD Models



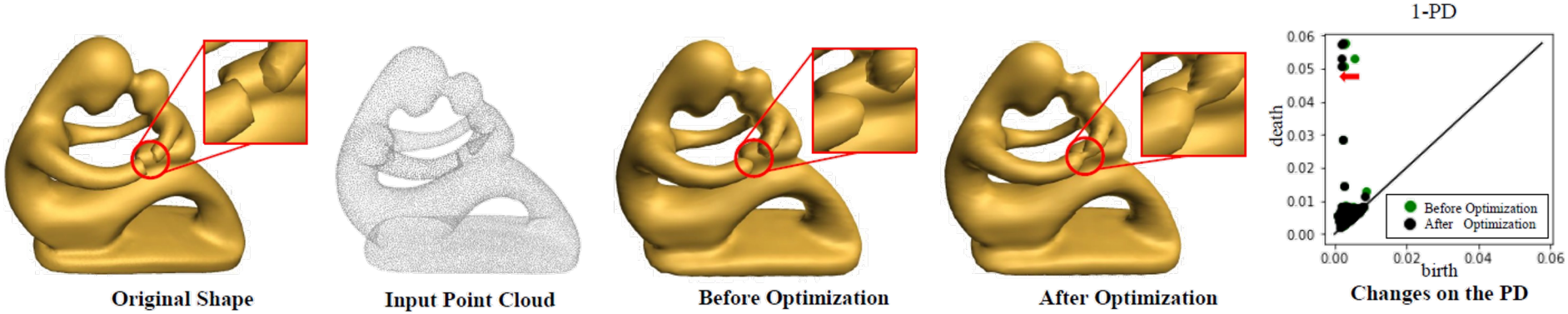
Topology-controllable implicit surface reconstruction



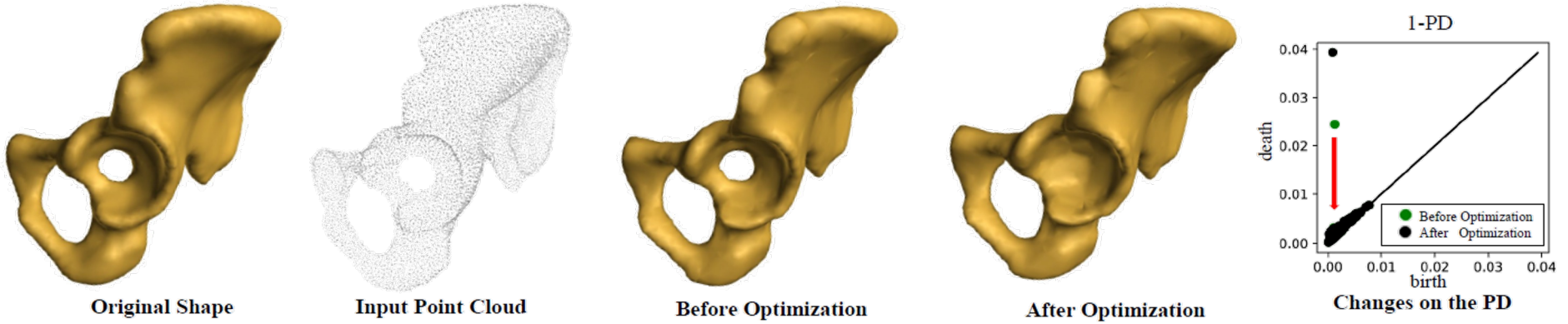
Zhetong Dong, Jinhao Chen, Hongwei Lin. Topology-controllable Implicit Surface Reconstruction Based on Persistent Homology. Computer-Aided Design,150: 103308 (2022)

求是创新

Topology-controllable implicit surface reconstruction



(a) Topology repair.

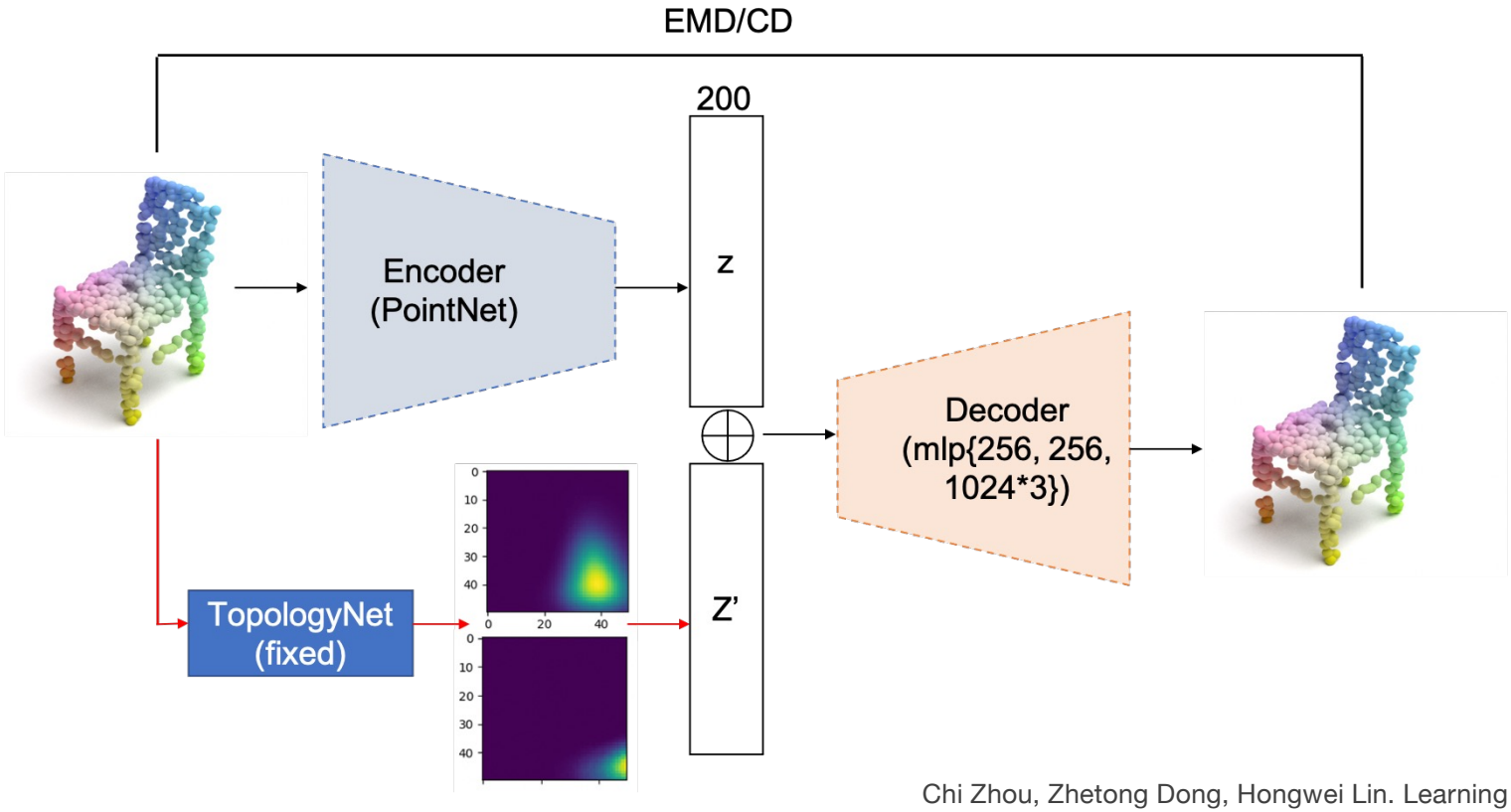


(b) Hole removal.

求是創新

Topology autoencoder-Generating point cloud based on topology

Topology autoencoder: Topology signal as extra input

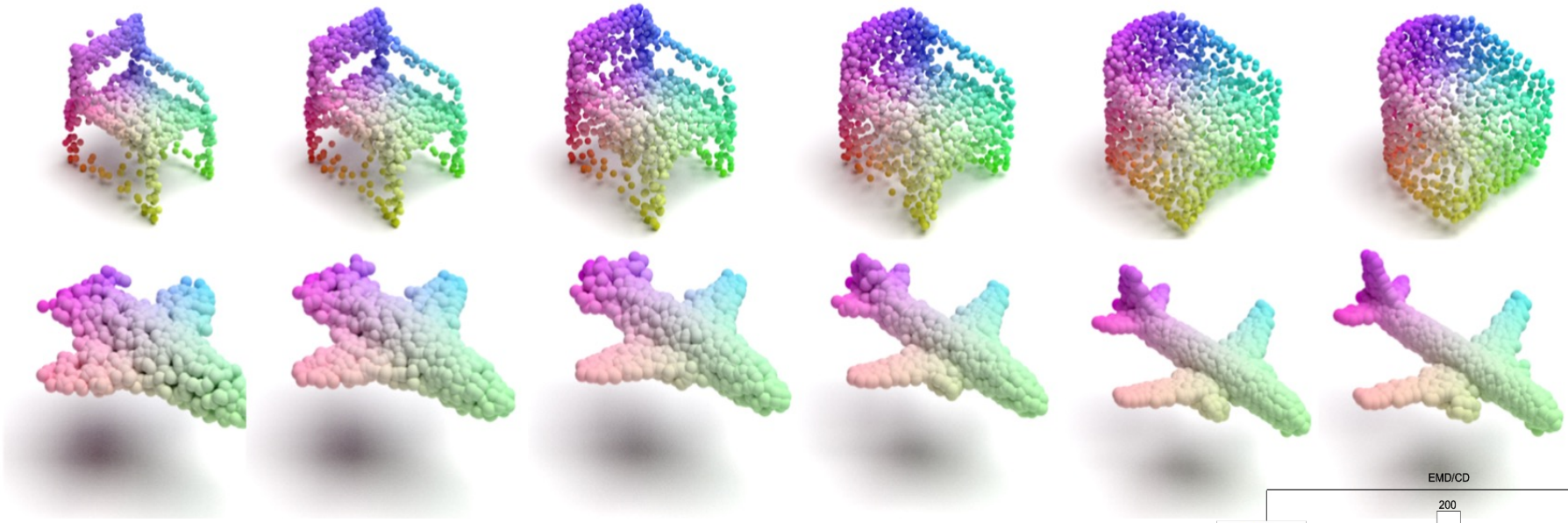


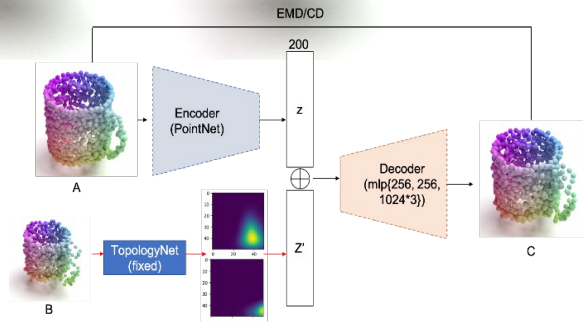
Chi Zhou, Zhetong Dong, Hongwei Lin. Learning persistent homology of 3D point clouds. Computers & Graphics, 102: 269-279, 2022

Topology autoencoder-Point cloud interpolation

Interpolation Results:

- The left most and right most are the input point clouds
- The middle point clouds are generated by interpolation using topological implicit vector





Chi Zhou, Zhetong Dong, Hongwei Lin. Learning persistent homology of 3D point clouds. Computers & Graphics, 102: 269-279 (2022)

神经网络可解释性：Functional Network

- 动机：利用持续同调，研究神经网络的结构和功能之间的关系
- 提出了神经网络的功能网络概念
- 通过研究功能网格的拓扑结构，可以更好理解神经网格的结构和功能之间的关系

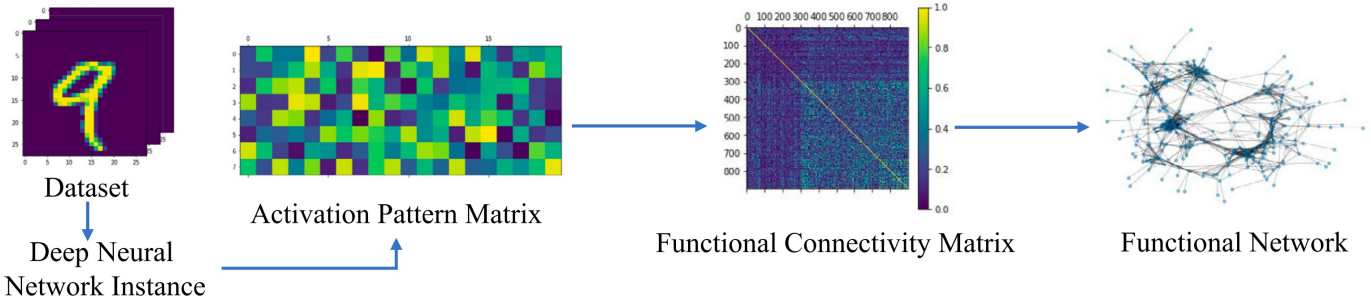
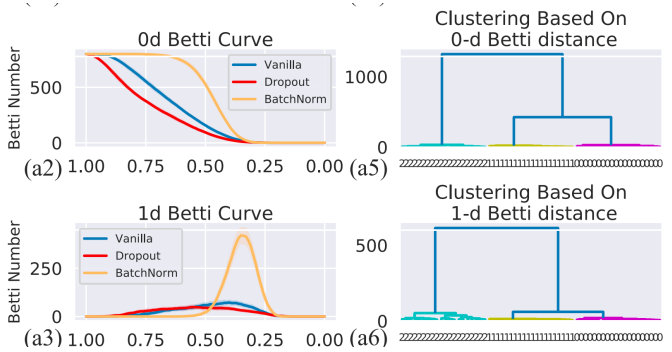
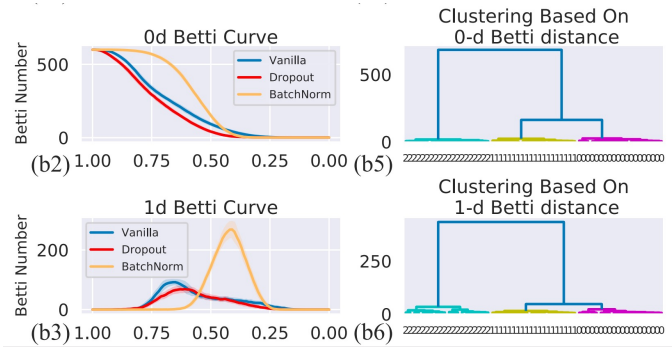


Fig. 1. Flow chart of constructing the functional network for a given deep neural network.



Fashion MNIST [300, 300]



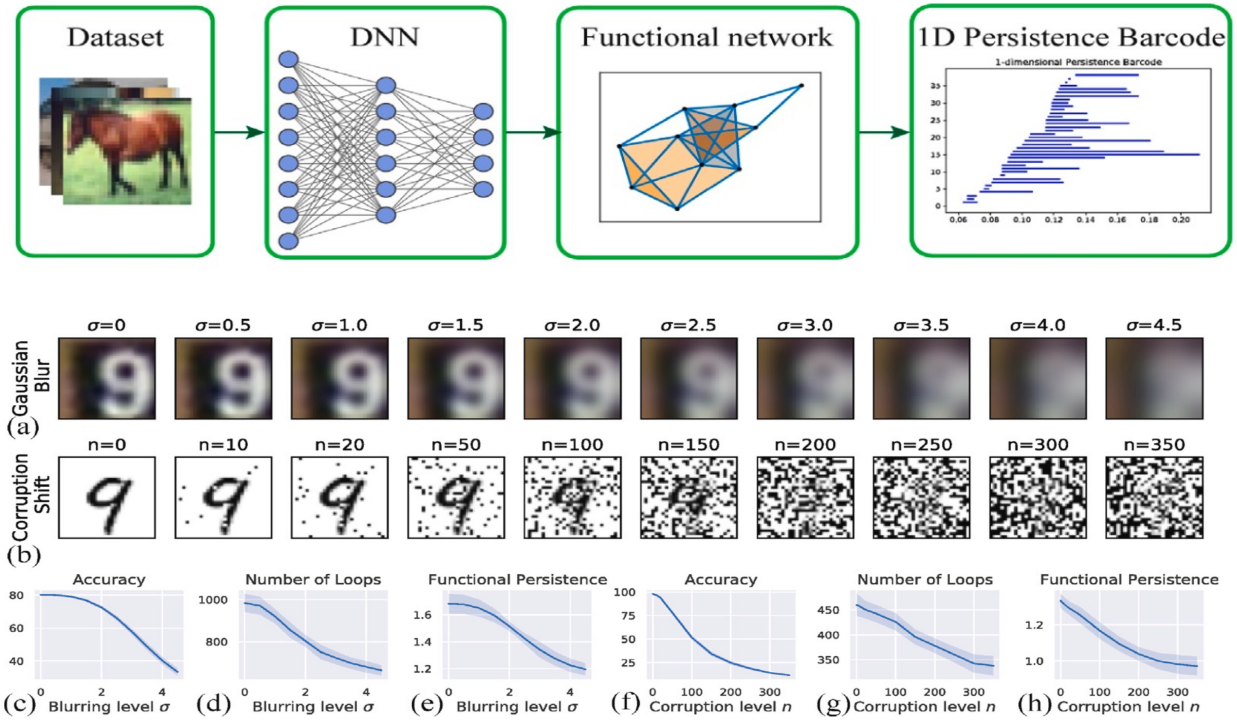
MNIST [300, 300]

Influence of different regularizations

Ben Zhang, Zhetong Dong, Junsong Zhang, Hongwei Lin. Functional network: A novel framework for interpretability of deep neural networks. Neurocomputing 519: 94-103 (2023)

神经网络的可解释性：Functional Loops

- 揭示了功能网络中的一维环结构（功能环）与样本中的信息量密切相关



- 构造深度学习网络的功能网络
- 研究功能网络的一维持续环与深度学习网络的功能复杂性之间的关系

- 功能环的个数、持续时间随着样本信息量的下降而减少
- 功能环的个数与持续时间可以反映深度学习网络学习到的特征量的多少

Ben Zhang, Hongwei Lin. Functional loops: Monitoring functional organization of deep neural networks using algebraic topology. Neural Networks 174: 106239 (2024)

求是创新

神经网络的可解释性：Topology structure optimization of reservoirs using GLMY homology

- 储层网络是时间序列处理的有效网络，它的主要成分是一个有向图
- 持续GLMY同调是分析有向图的有力工具
- 发现并验证了储层网络的性能与一维持续GLMY同调群有密切关系
- 基于一维持续GLMY同调群的生成元，发展了储层网络结构优化方法，显著提升了储层网络的预测能力

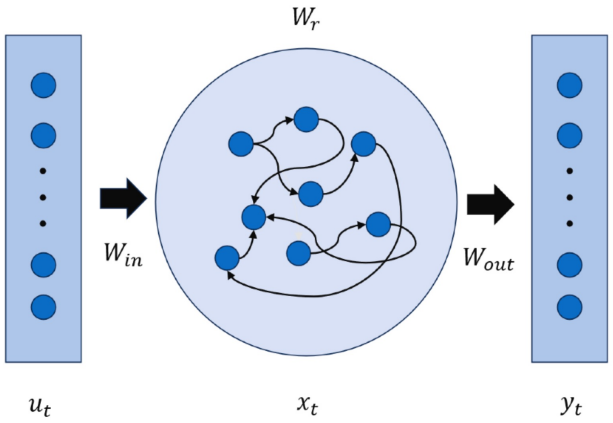


Fig. 3. Structure of reservoir computing model.

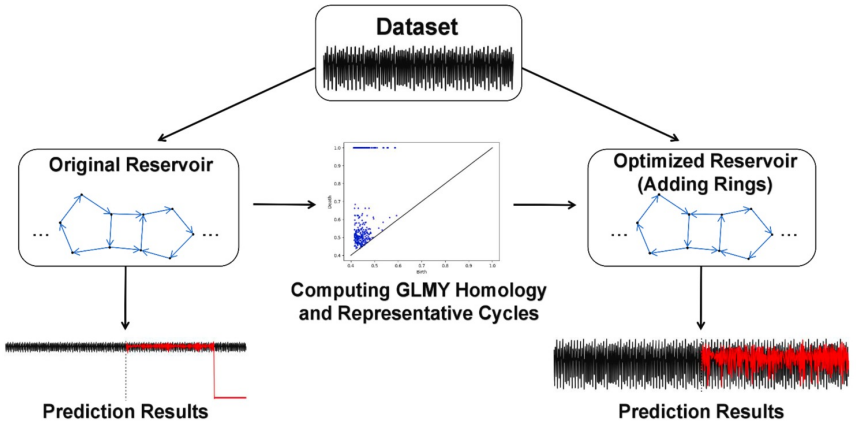


Fig. 1. Illustration of the flowchart of our method and experimental procedures.

Yu Chen, Shengwei Wang, Hongwei Lin. Topology structure optimization of reservoirs using GLMY homology. Neural Networks 202: 109004 (2024)

05

总结



总结

浙江大学 ZHEJIANG UNIVERSITY

- **计算机拓扑设计：拓展几何造型的应用范围**
  - 将计算拓扑与传统CAD深度融合，从单纯的几何元素堆砌转向“拓扑骨架”驱动的设计
  - 设计师可直接定义与操控模型的连通性、孔洞数量等深层拓扑特征，为复杂结构生成提供了数学化的底层逻辑支撑
- **AI+CAD：催生高层语义建模技术**
  - 传统AI仅能执行基础几何命令，而融合计算拓扑与AI后，模型被转化为可被机器理解的属性拓扑图
  - 这让AI具备了“读懂”设计意图的能力，能够精准识别特征的工程语义，实现从“执行指令”到“理解设计”的质变
- **具身智能：为具身智能装上“拓扑眼睛”**
  - 计算拓扑为具身智能开辟了全新的感知维度，让机器人超越像素与几何的浅层感知，理解环境的连通性、孔洞与结构关联
  - 这种“拓扑感知”对光照变化、几何细节扰动及部分遮挡具有极强的鲁棒性，是机器人适应非结构化真实世界的关键

关于计算机辅助拓扑设计的更详细信息，请浏览主页：
person.zju.edu.cn/hwlin

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谢谢！

浙江大学-几何与拓扑计算