Computer Aided Topological Design

—Applications of computational topology in geometric design and processing

Hongwei Lin
School of Mathematical Science, Zhejiang University
Nov. 22, 2024

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SEEKING TRUTH
PURSUING INNOVATION

Brief History

- **Computational Topology** is first presented in:

  Dey T K, Edelsbrunner H, Guha S. Computational topology. Contemporary mathematics, 1999, 223: 109-144.
- **Computational Geometry → Computational Topology**
- **Topological Data Analysis:** Proposed in 2009

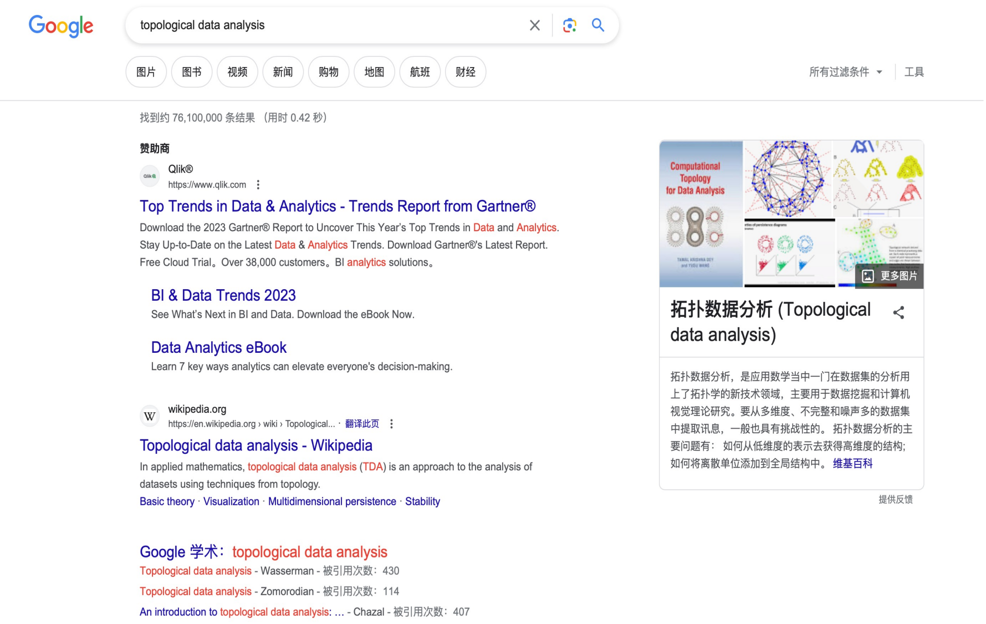
  Carlsson G. Topology and Data. Bulletin of the American Mathematical Society, 2009, 46(2): 255-308.
- **Computer Aided Geometric Design → Computer Aided Topological Design**

1.2 创始人介绍

图：Ayasdi创始人Gurjeet Singh、Gunnar Carlsson、Harlan Sexton（从左往右）

2008年，Gunnar Carlsson、Gurjeet Singh和Harlan Sexton在斯坦福大学进行了12年的研发工作后联合创立了Ayasdi公司。在斯坦福大学研究期间，三位创始人获得了美国国防部高级研究计划局（DARPA）和美国情报高级研究计划署（IARPA）的资助，用于“高风险、高回报”的研究项目。2012年，Ayasdi获得了由Floodgate Capital和Khosla Ventures牵头的1025万美元的首轮融资，就此这家高精技术公司开启了它的征程。

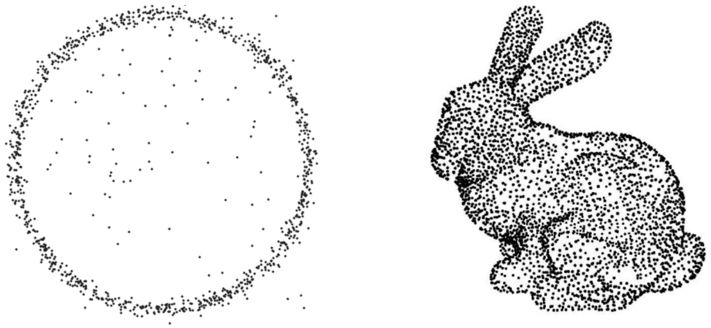
公司总顾问Gunnar Carlsson是世界上最著名的数学家之一，他拥有哈佛大学的本科学位和斯坦福大学的博士学位，在过去的35年里，先后在芝加哥大学、加州大学、普林斯顿大学任教。自1991年以来，Gunnar一直在斯坦福大学担任数学教授，他是斯坦福大学数学领域里拓扑学研究的思想领袖。在21世纪初，这项工作获得了美国国家科学基金会(NSF)和美国国防部高级研究计划局（DARPA）1000万美元的研究拨款，用于研究拓扑数据分析技术(TDA)在美国政府关切问题上的应用。



https://www.weiyangx.com/379809.html

TDA已经成功应用于金融、生物、医学、药物设计、人工智能、脑神经科学等领域

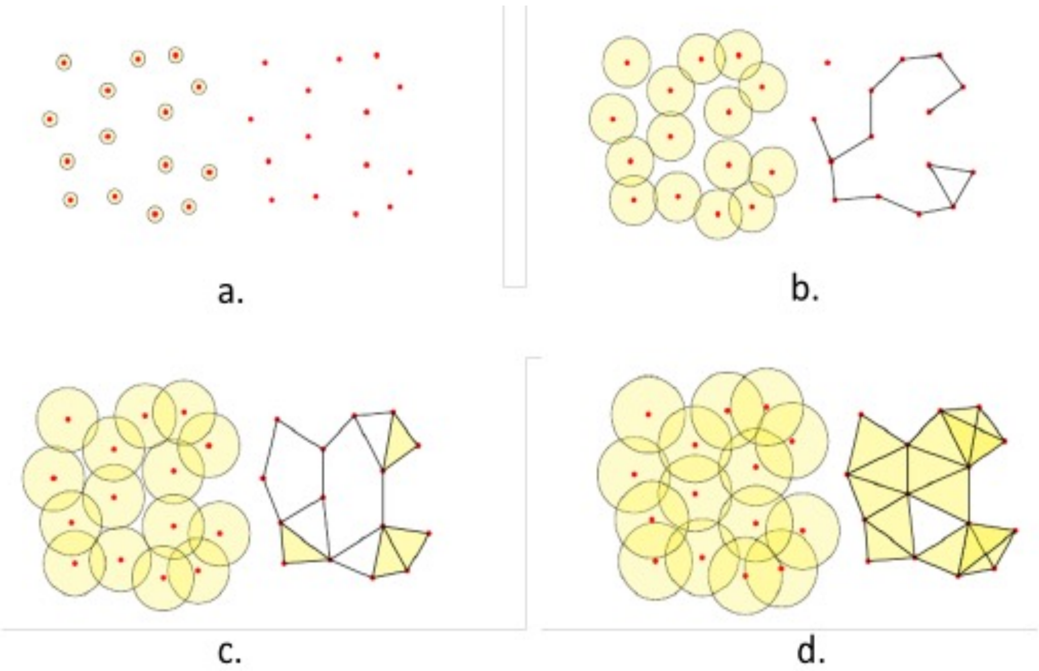
Motivation of Topological Data Analysis



Topological Inference for point cloud data

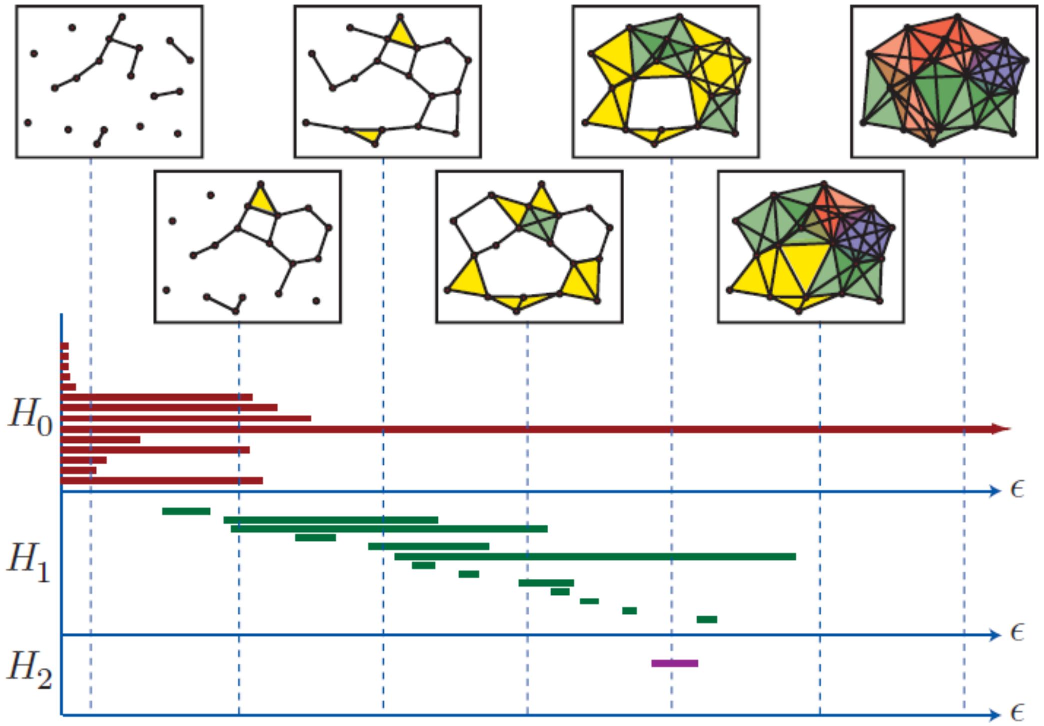
- How to **infer** the **essential topological features** of a hidden shape?
  - To extract components, loops, voids and higher dimensional features
  - To measure their importance
  - Stable with respect of small perturbations
- Mathematical Background
  - Algebraic Topology (Homology Theory)
  - Statistics
  - Geometry
- A promising bridge between topology and geometry in the view of computation

Filtration，Persistent Homology



Filtration (Čech complex)

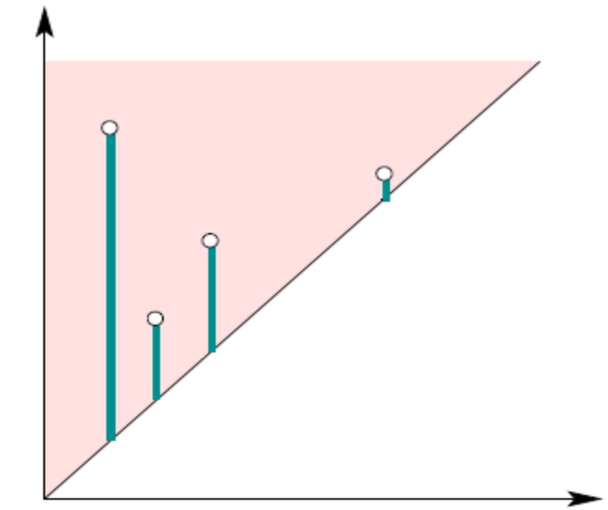
As the radii of open balls grow, the **simplicial complex** can be constructed on point cloud data, which gives a nested complex sequence, a **filtration**.



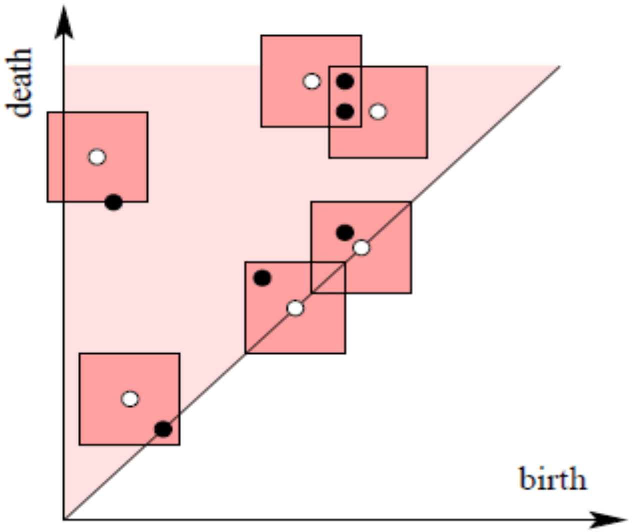
Barcode

- As time goes by, the radii of open balls grow
- New k-holes are **born**
- Some k-holes are destroyed by higher-dimensional simplex and **die**

Persistence Diagram, Distance and Stability

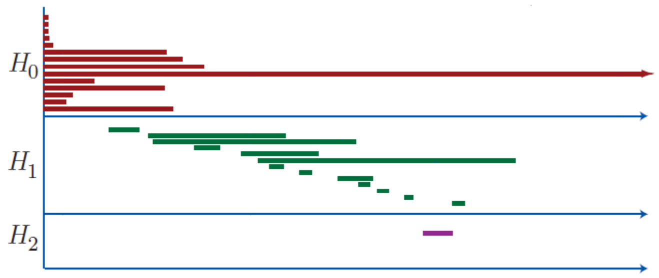


Persistence Diagram



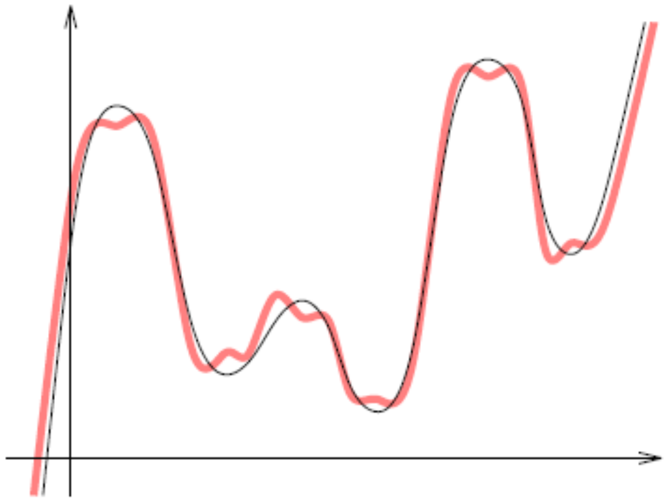
$$W_\infty(X,Y) = \inf_{\eta:X\to Y} \sup_{x\in X} \|x-\eta(x)\|_\infty$$

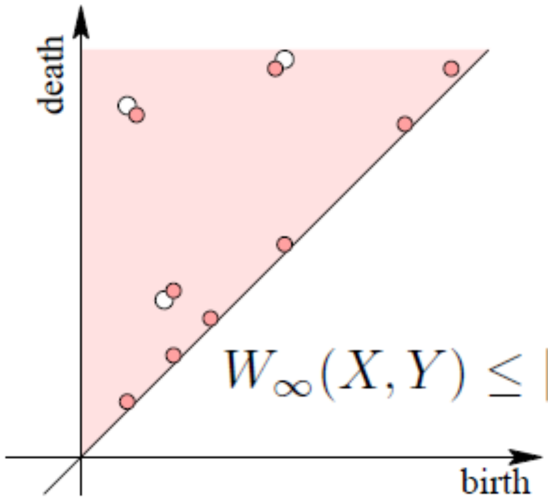
Wasserstein Distance



Barcode

Stability: The ***Wasserstein distance*** of two persistence diagrams is controlled by the small perturbations on the tame function f



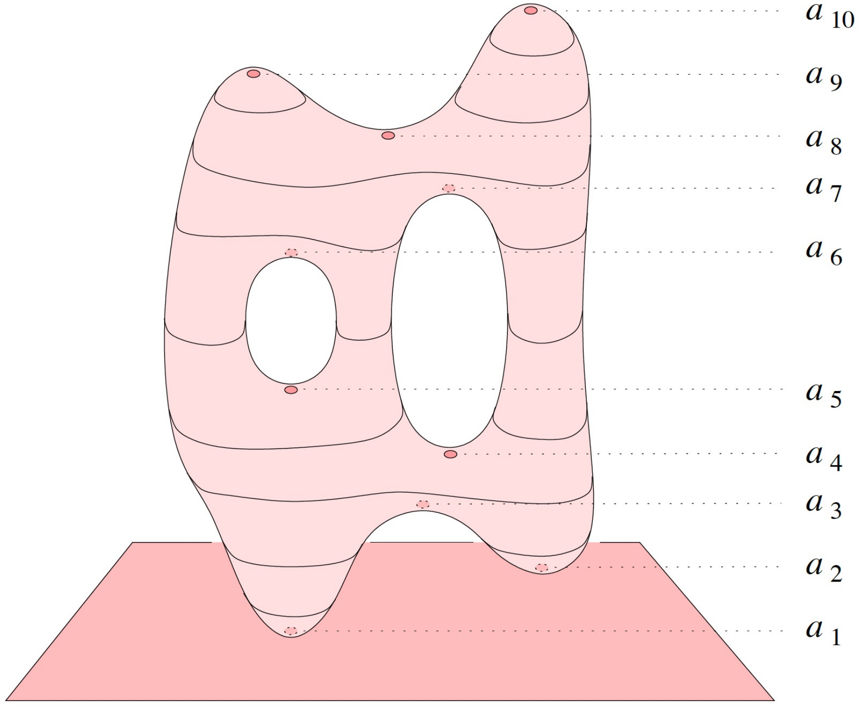


$$W_\infty(X,Y) \leq \|f-g\|_\infty$$

Computational Topology

- Distance definition and computation
- Stability
- Representation
- Extended persistence
- Zigzag persistence
- Multiparameter persistence
- Persistence Module

…



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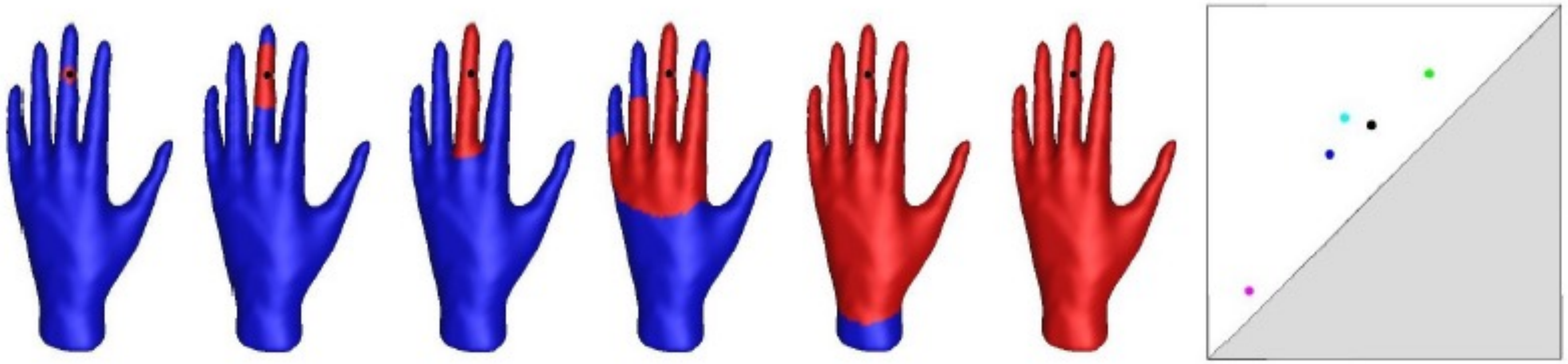
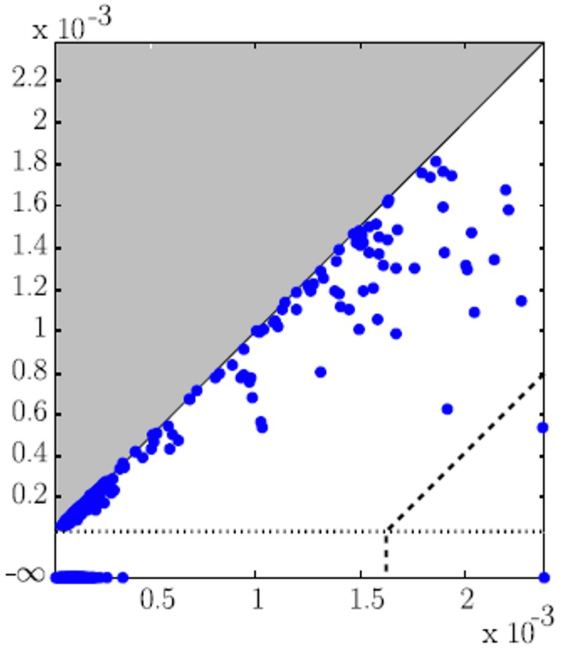
Applications: Shape Description



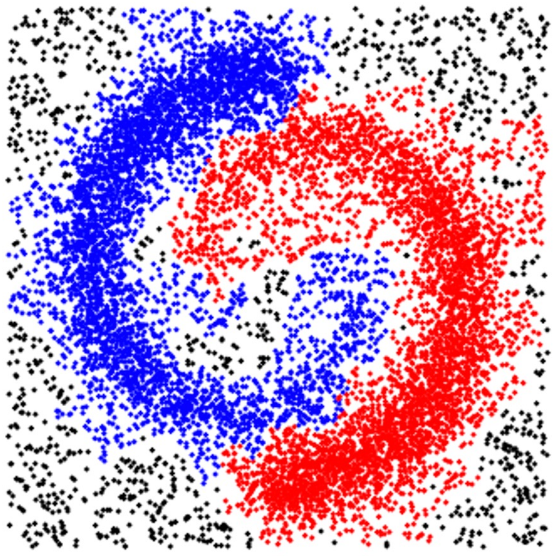
- The persistence diagram corresponding to this family is shown in the right
- The pink, blue, light blue, black and green points correspond to the middle, index, ring, pinky and thumb respectively

Carrière M, Oudot S Y, Ovsjanikov M. Stable topological signatures for points on 3d shapes. Computer graphics forum. 2015, 34(5): 1-12.

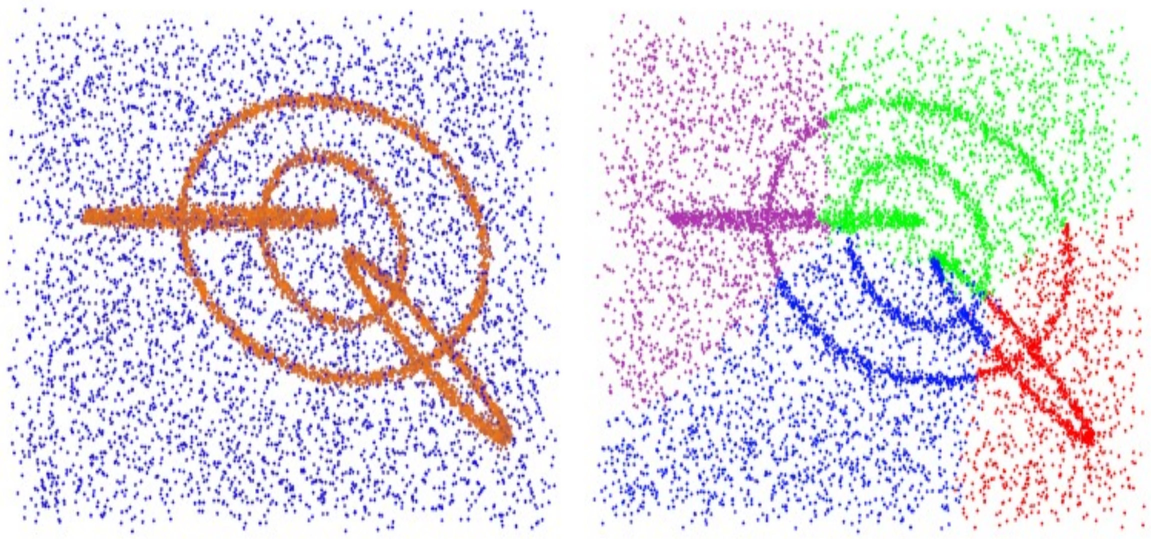
Applications: Clustering of Point Clouds



(a)



(b)



- The rings are detected by the clustering method with persistence diagram (left)
- Compared with the result obtained by spectral clustering (right)

Chazal F, Guibas L J, Oudot S Y, et al. Persistence-based clustering in Riemannian manifolds[J]. Journal of the ACM (JACM), 2013, 60(6): 1-38.

Applications: Topological Denoising

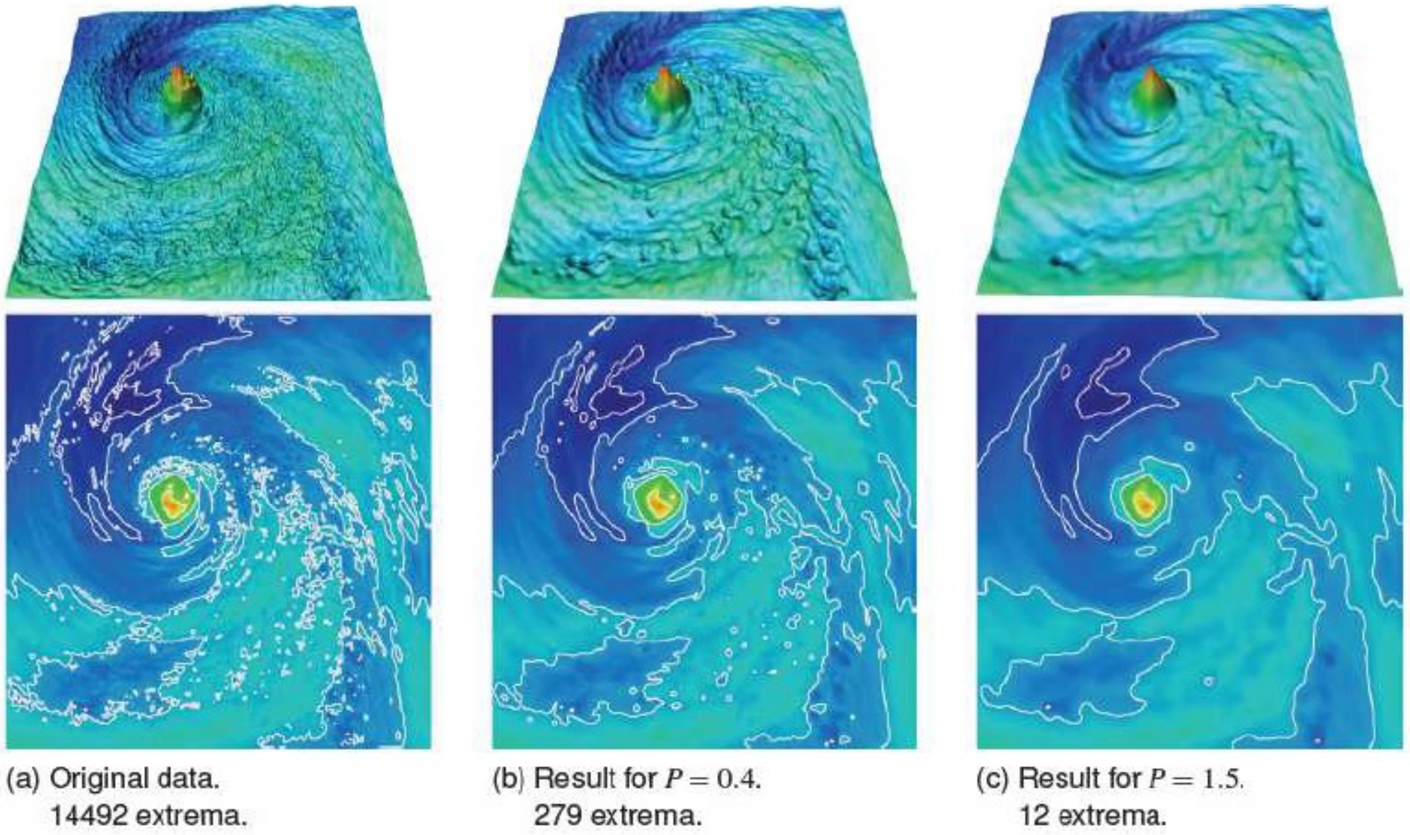


Figure 10. Temperature in the Hurricane Isabel data set (slice $z = 20$). Using persistence-based filtering, we create a hierarchy of scalar fields: with increasing persistence P, our method creates increasingly smoother versions of the data.

- Keep the salient features while denoising by persistence-based filtering

Günther D, Jacobson A, Reininghaus J, et al. Fast and memory-efficienty topological denoising of 2D and 3D scalar fields. IEEE transactions on visualization and computer graphics, 2014, 20(12): 2585-2594.

Applications: Topology-aware Reconstruction

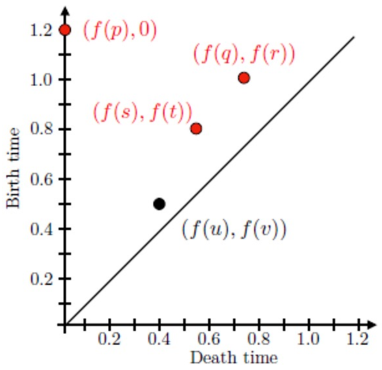
- **Objective Function:**

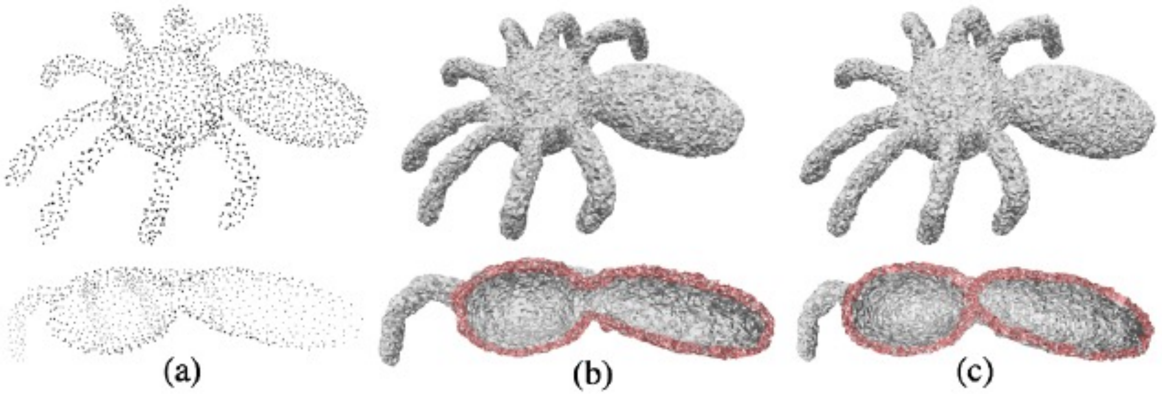
$$-((d_1 - b_1)^2 - (d_2 - b_2)^2)$$

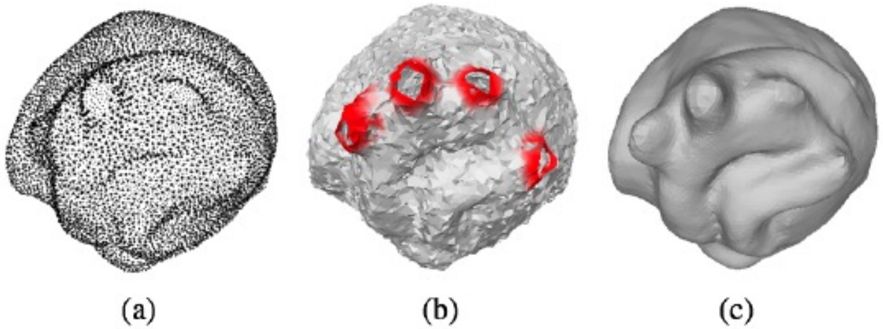
- The persistence of the first point is getting longer and longer
- The persistence of the other points are getting shorter and shorter.

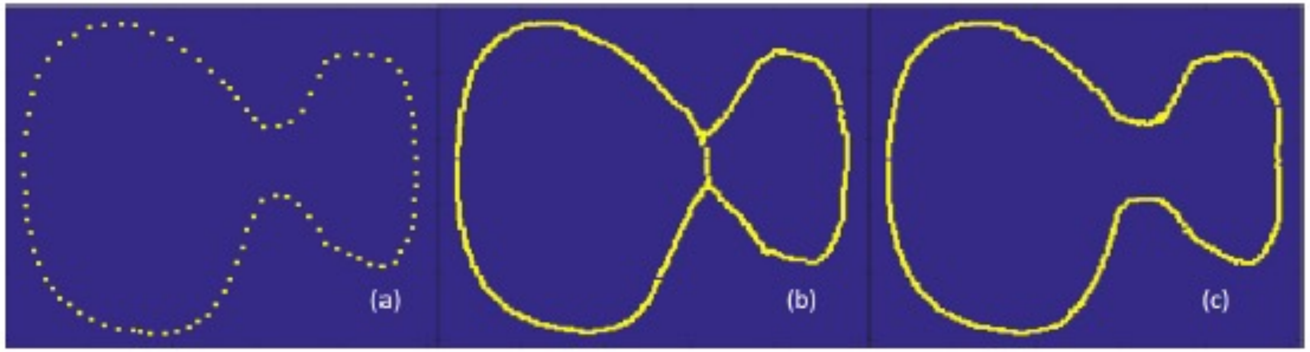
$$-((d_2 - b_2)^2 - (d_3 - b_3)^2)$$

- The persistence of the first two point is getting longer and longer
- The persistence of the other points are getting shorter and shorter



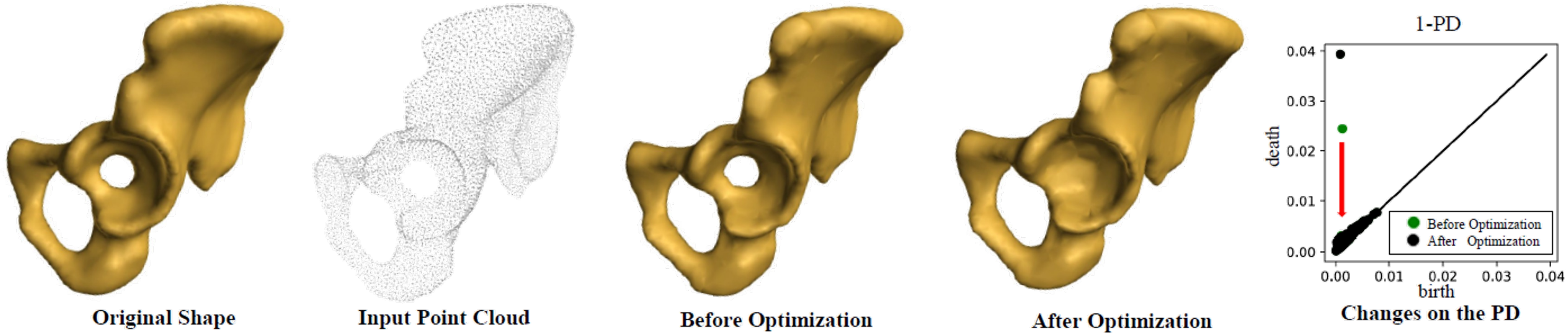






Brüel-Gabrielsson, Rickard, et al. Topology-Aware Surface Reconstruction for Point Clouds. *Computer Graphics Forum*. Vol. 39. No. 5. 2020.

Applications: Topology-controllable implicit surface reconstruction



(b) Hole removal.

Application: Hole removal

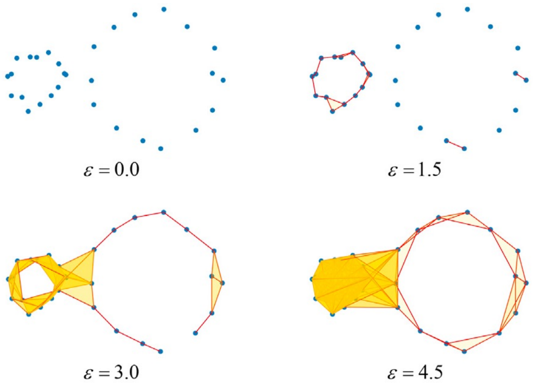
Motivation: To remove the 1D holes (1D homology classes) on the shape

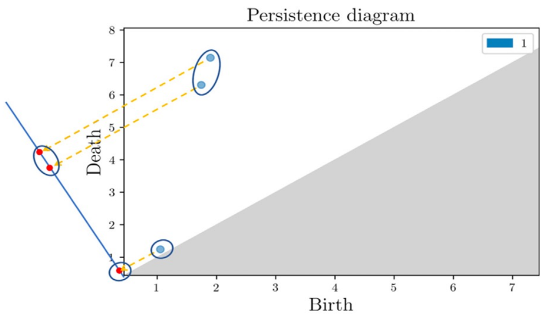
Topology-controllable optimization: To minimize $L^{(1)} = d_2 - b_2$, where (b_2, d_2) represents the persistence pair of the second most persistent feature

Result: The extracted surface was obtained with the removed small-scale handle loop

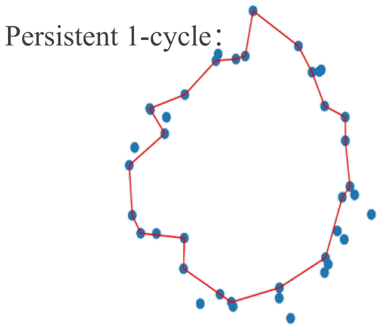
Zhetong Dong, Jinhao Chen, Hongwei Lin. Topology-controllable Implicit Surface Reconstruction Based on Persistent Homology. Computer-Aided Design,150: 103308, 2022 (SPM 2022)

Topology understanding：

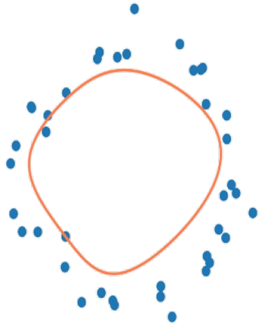




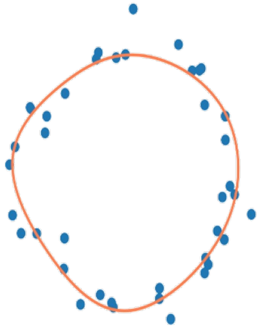
(a) Input point cloud.



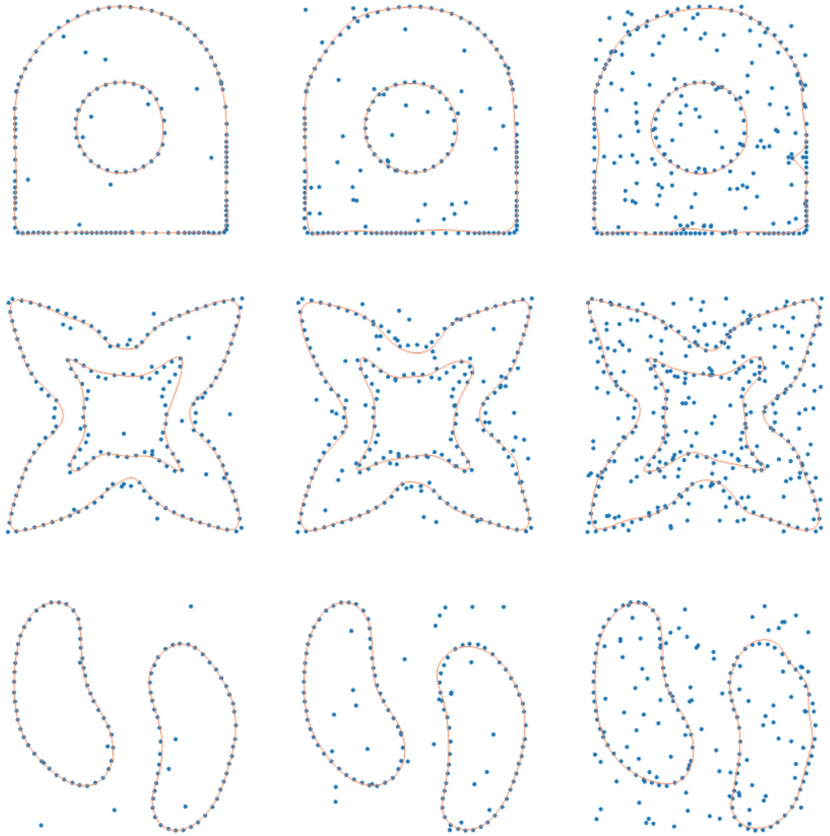
(b) Extracted persistent 1-cycle.



(c) Initial parametric B-spline curve.



(d) Reconstructed parametric B-spline curve.



(d) Points with 10% added outliers. (e) Points with 30% added outliers. (f) Points with 100% added outliers.

Yaqi He, Jiacong Yan, Hongwei Lin. Robust reconstruction of closed parametric curves by topological understanding with persistent homology. Computer-Aided Design, 165: 103611, 2023

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SEEKING TRUTH
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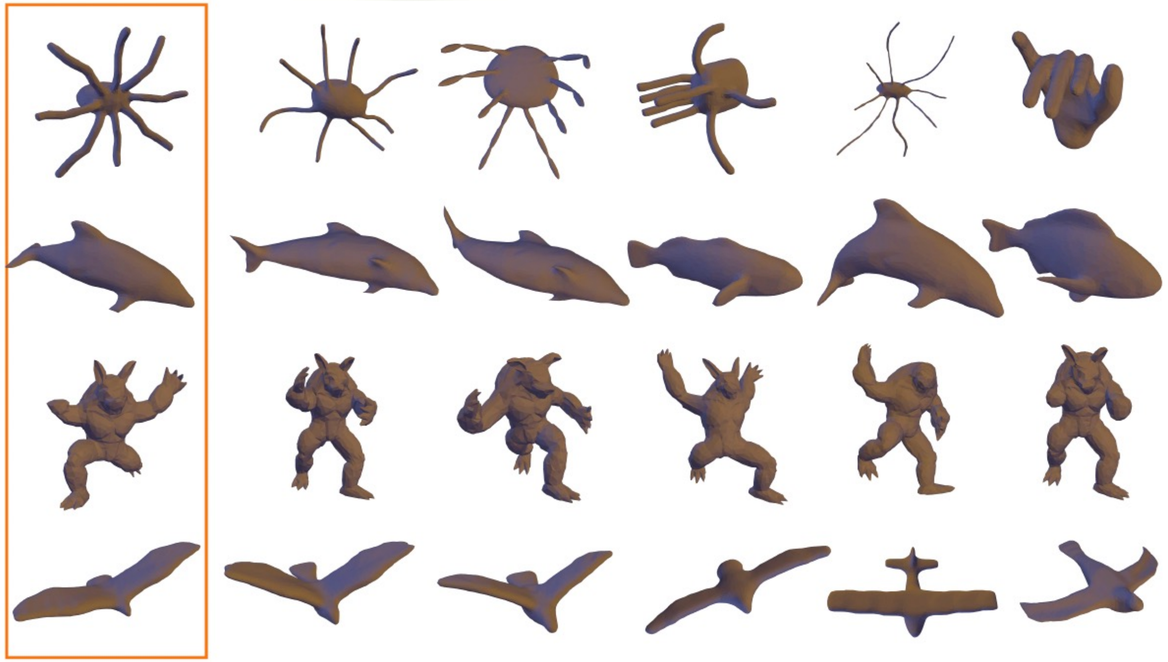
Introduction

- **Porous Retrieval:** Given a query (3D model), retrieving models most similar to the query in a model base
- **Porous Design:** Keep connectivity of the designed porous
- **Porous Printing:** Guarantee topology consistency between design model and printing model

Query

Top 5 matches



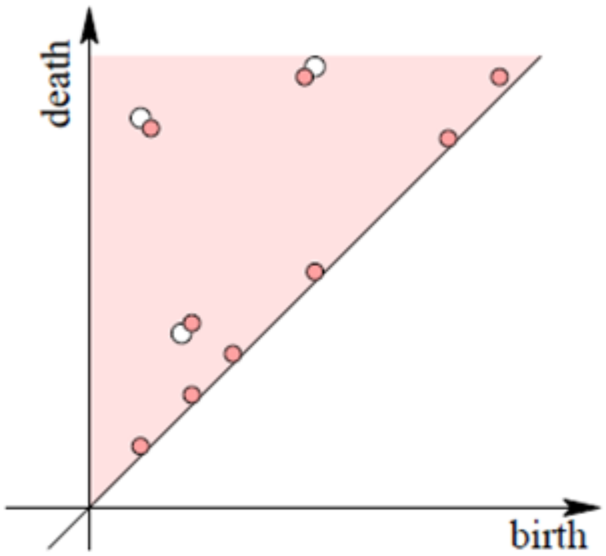
Computational topology is possible to be the theoretical foundation for the design and processing of porous structure

e

https://youtu.be/dLTKNY1ZzAM?si=Fy4e6VWl6BYCpIqY

Vectorizing Representation for Porous Retrieval

- **Analysis and applications of PDs**
  - To extract more statistical information from PDs
  - To learn from PDs via machine learning approaches
  - To design feature descriptors using PDs
- **Space of PD with metric W_p**
  - Irregular points on a PD
  - Inefficient to compute the W_p distance



PD should be transformed into a proper representation.

$$W_p(PD^{(1)}, PD^{(2)}) = \inf_b \Big(\sum_{u \in PD^{(1)}} \|u - b(u)\|_\infty^p \Big)^{1/p}$$

Vectorizing Representation of a PD: to map a PD into a Hilbert space.

Representation: Persistence Images

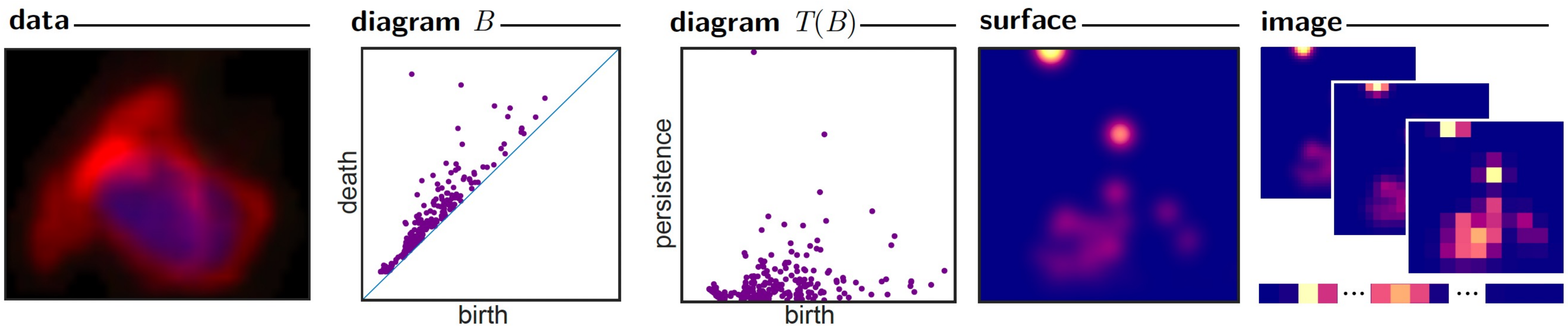
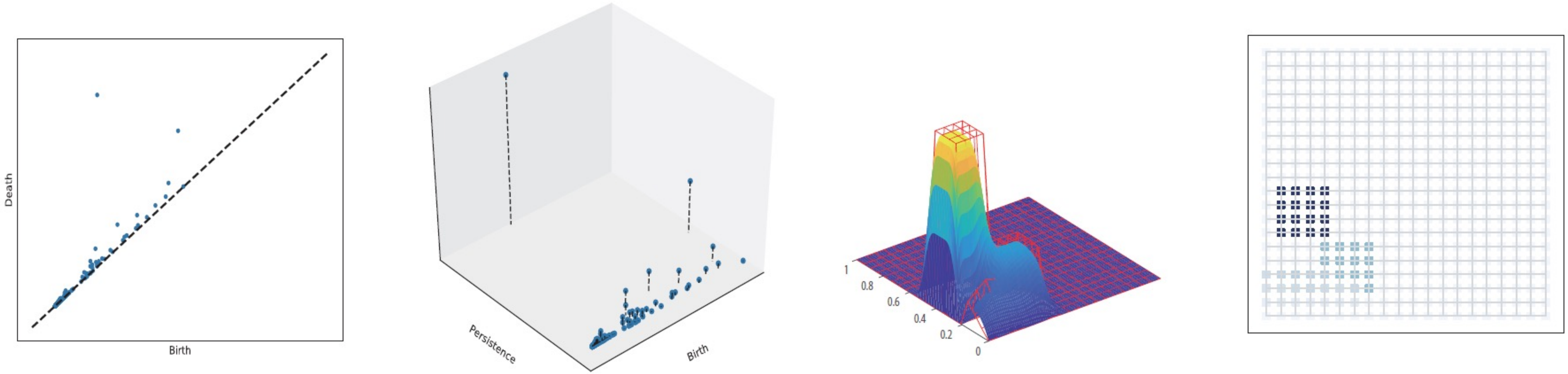


Figure 1: Algorithm pipeline to transform data into a persistence image.

- The persistence image (PI) provides a vector representation of the information in a persistence diagram
- The Gaussian function is chosen at each point in the transformed persistence diagram $T(B)$
- It makes the results of persistent homology available in machine learning algorithms such as linear SVM

Adams H, Emerson T, Kirby M, et al. Persistence images: A stable vector representation of persistent homology. Journal of Machine Learning Research, 2017, 18(8): 1-35.

Vectorization: Persistence B-spline Grid (PBSG)



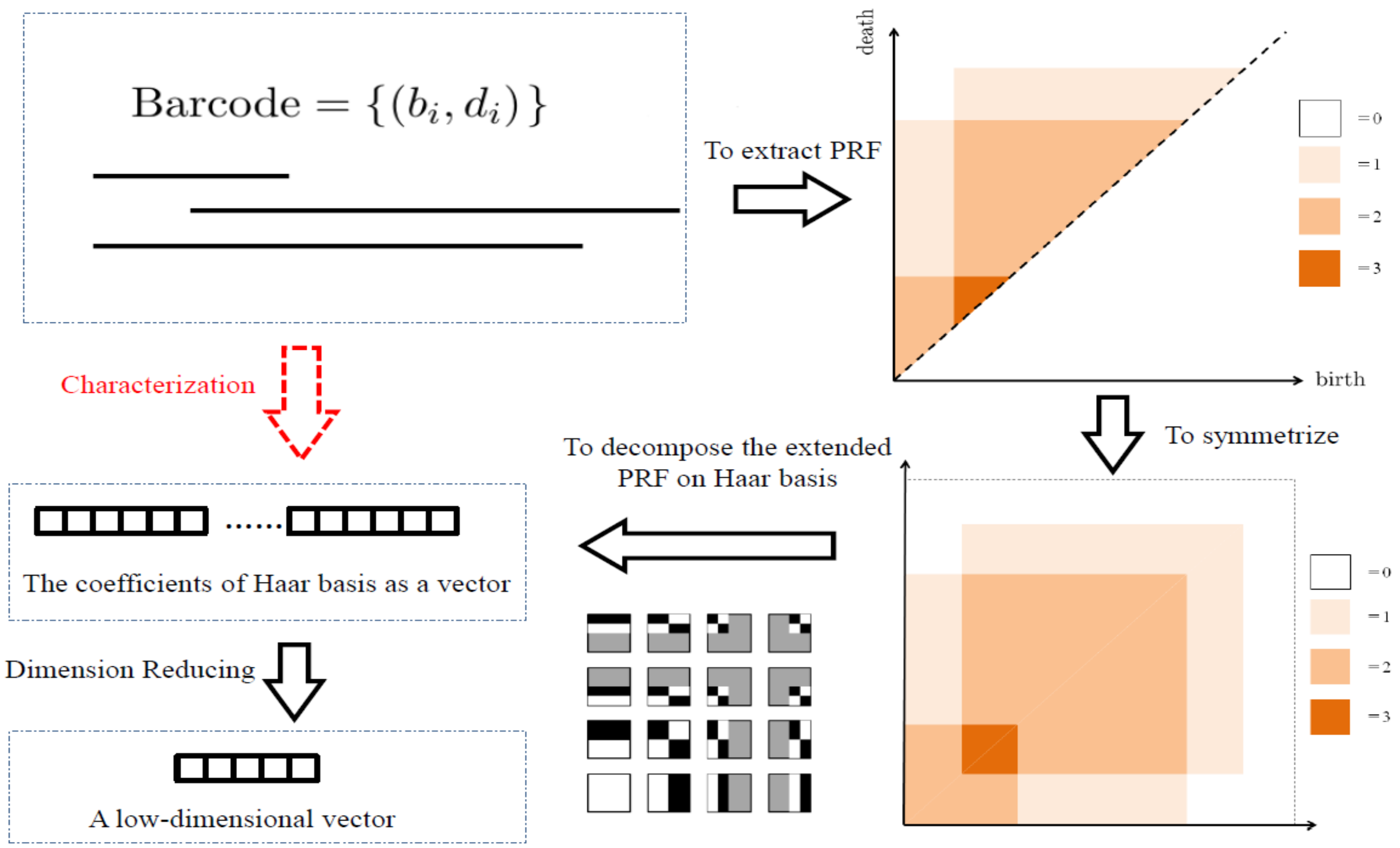
- Transform a PD into the birth-persistence coordinates and assign a value to each point in a PD according to its importance
- Fit the data points in 3D by a cubic uniform B-spline surface
- The control grid is obtained to generate a vector by concatenating rows of z-coordinates of the control grid
- **Persistence B-spline Grid** is defined by the matrix formed by z-coordinates of the control points

PBSG performs better than PI in most cases

Zhetong Dong, Hongwei Lin, Chi Zhou, Ben Zhang, Gengchen Li. Persistence B-Spline Grids: Stable Vector Representation of Persistence Diagrams Based on Data Fitting. Machine Learning, 113(3): 1373-1420, 2024

Characterization of Persistence Barcode by Haar Basis

- Transform Barcode to extended PRF
- Wavelet decomposition on extended PRF
- Generate shape descriptor by dimension reduction to the wavelet decomposition coefficients



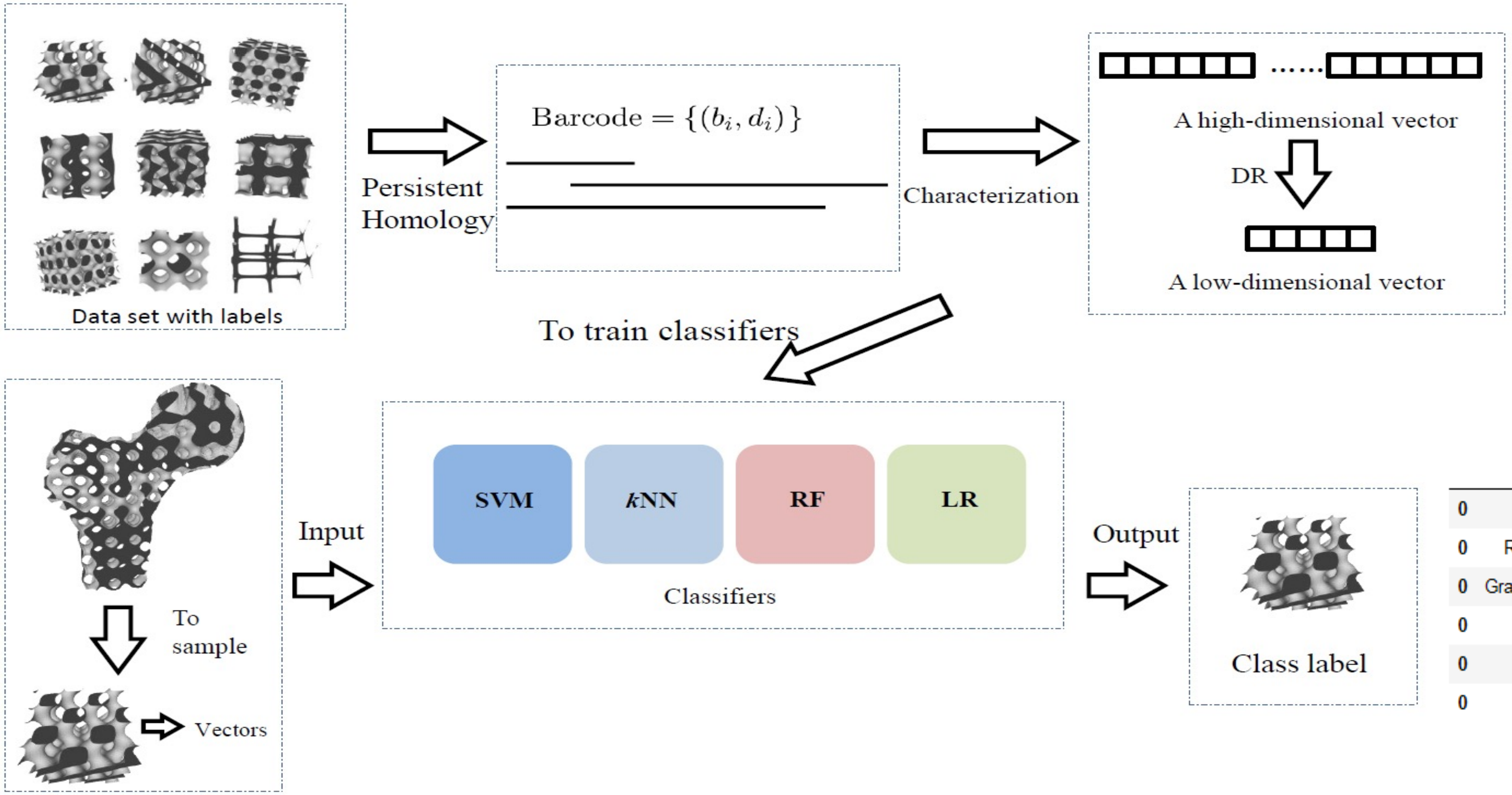
- Given Barcode $\{(b_i, d_i) | 0 \le b_i \le d_i, i \in I, |I| < \infty\}$
- Extended persistent rank function is defined as:

$$\tilde{r}_i(s,t) = \begin{cases} 1, & b_i \le s, t \le d_i \\ 0, & otherwise \end{cases}$$

Zhetong Dong, Chuanfeng Hu, Chi Zhou, Hongwei Lin, Vectorization of persistence barcode with applications in pattern classification of porous structures , Computers & Graphics, Volume 90, 2020,Pages 182-192

Characterization of Persistence Barcode by Haar Basis

浙江大學
ZHEJIANG UNIVERSITY

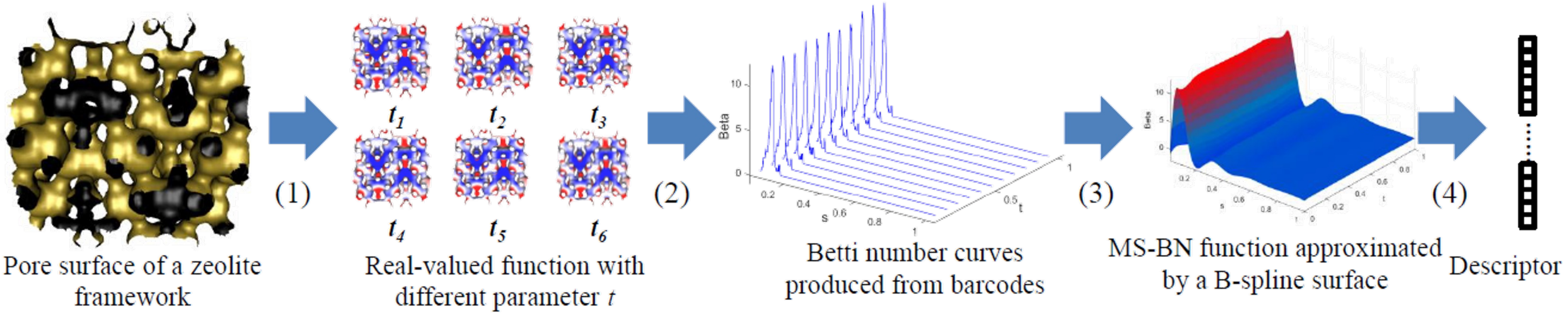


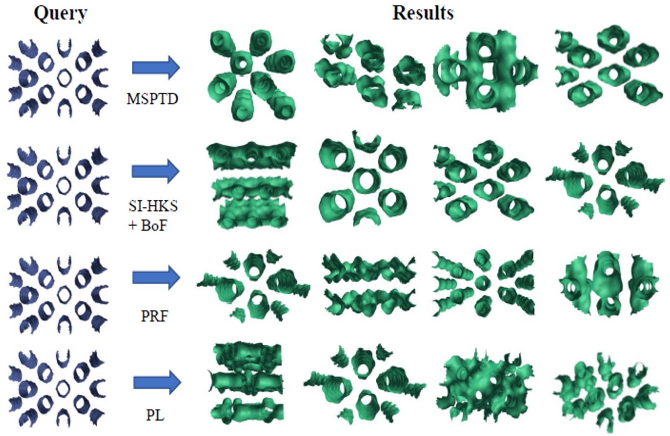
	Classifier	Accuracy
0	KNeighborsClassifier	0.990000
0	RandomForestClassifier	0.986111
0	GradientBoostingClassifier	0.985000
0	LogisticRegression	0.982222
0	LinearSVC	0.981667
0	SVC	0.990000

Zhetong Dong, Chuanfeng Hu, Chi Zhou, Hongwei Lin, Vectorization of persistence barcode with applications in pattern classification of porous structures , Computers & Graphics, Volume 90, 2020,Pages 182-192

Multiscale Persistent Topological Descriptor

- Calculating a series of Betti number curves of the heat kernel on a porous surface at some times
- Generating the B-spline surface by lofting the Betti number curve sequence
- The control grid of the B-surface surface is taken as the shape descriptor



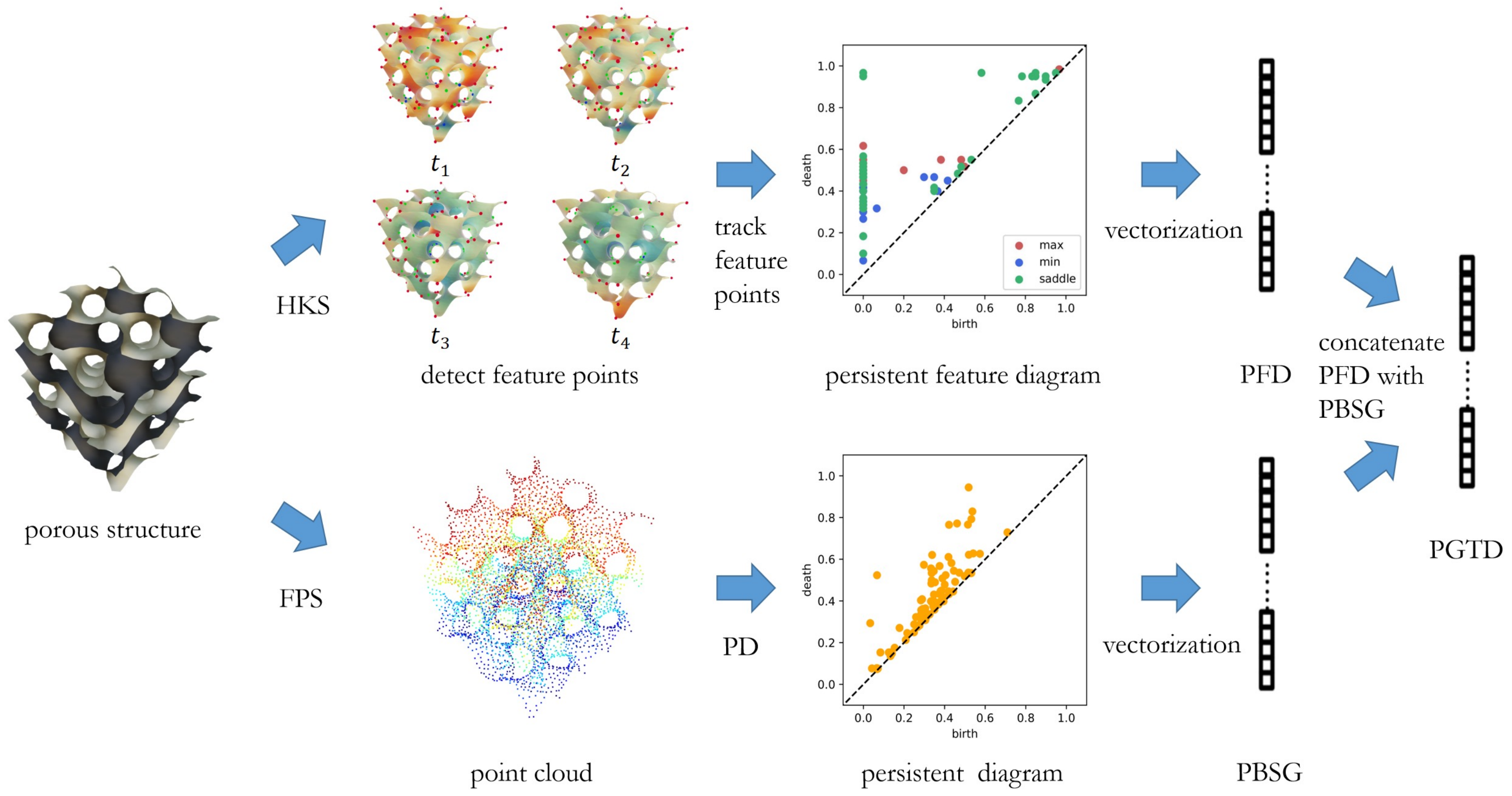


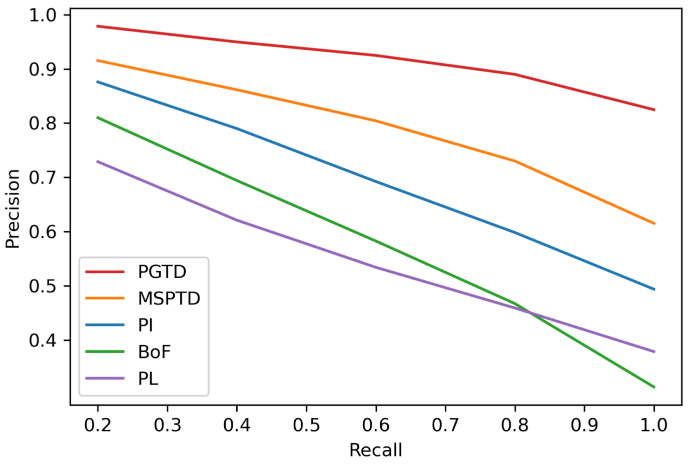
Methods/Accuracy (%)	NN	FT	ST	DCG
SI-HKS with histogram	90.1	73.3	91.6	92.7
SI-HKS with BoF	81.3	76.1	91.0	91.8
PL descriptor	68.1	59.3	75.0	83.4
PI descriptor	81.5	71.8	86.7	90.1
PRF descriptor	81.8	74.1	87.5	90.6
MSPTD (ours)	**94.9**	**78.4**	**94.5**	**95.5**

Zhetong Dong, Junyu Pu, Hongwei Lin. Multiscale Persistent Topological Descriptor for Porous Structure Retrieval. Computer Aided Geometric Design, 88: 102004 (2021) (SIAM-GD 2021)

Persistent Geometry-Topology Descriptor

- Geometric information is represented by persistent feature diagram (PFD), formed by tracing the emergence and disappear of the heat kernel feature points
- Topological information is represented by persistent diagram (PD)
- Persistent Geometry-Topology Descriptor is generated by concatenating the vectors of PFD and PD





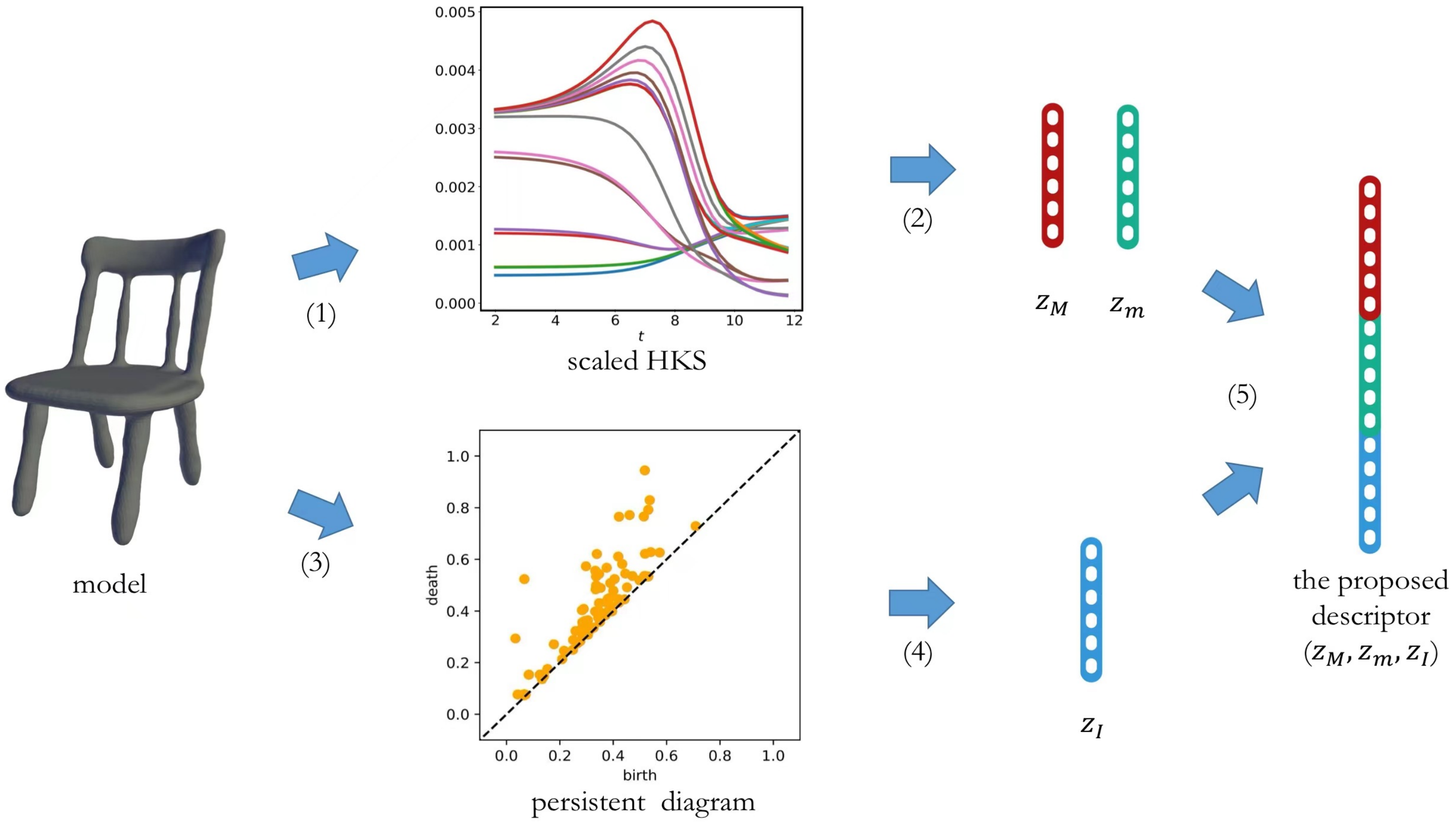
Methods/Accuracy (%)	NN	FT
MSPTD	86.8	71.0
HKS with BoF	75.1	50.8
PL descriptor	62.8	48.9
PI descriptor	81.6	61.8
PGTD (ours)	**96.3**	**86.5**

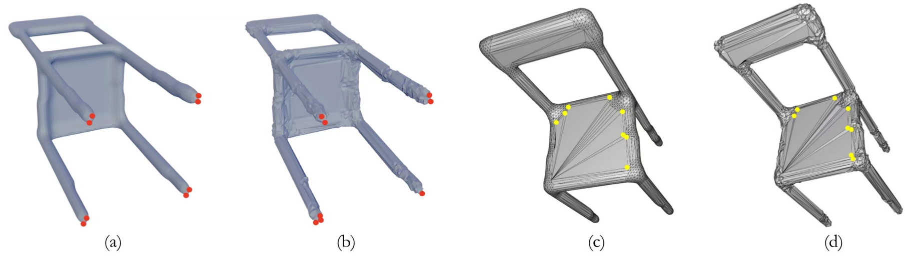
Peisheng Zhuo, Zitong He, Hongwei Lin. Persistent geometry-topology descriptor for porous structure retrieval based on heat kernel signature. Graphical Models, 133 (2024), 101219

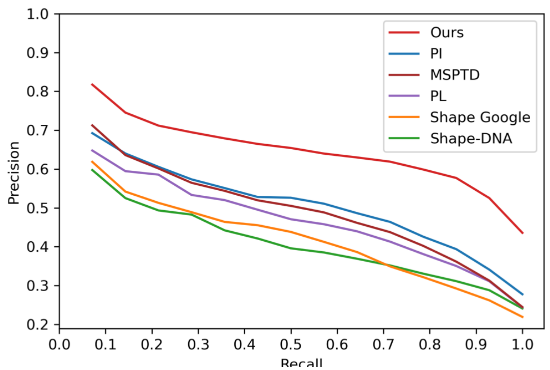
求是創新

Persistent Heat Kernel Signature Descriptor

- Calculate the heat kernel signature curve at each mesh vertex, as well as the maximums and minimums on the curves
- Shape descriptor is formed by first sorting the maximums and minimums, and then concatenating them with the vector of PD







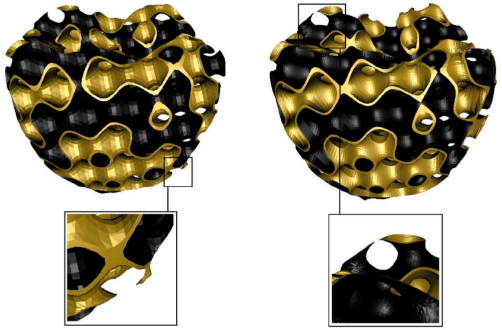
Methods/Accuracy (%)	NN	FT	ST
Shape-DNA (Reuter et al., 2006)	46.49	37.66	53.01
Shape Google (Bronstein et al., 2011)	52.63	37.34	52.57
PI (Adams et al., 2017)	61.40	44.67	62.47
PL (Bubenik, 2015)	57.89	40.85	58.02
MSPTD (Dong et al., 2021)	60.53	43.92	62.66
Our descriptor	**78.07**	**59.09**	**73.56**

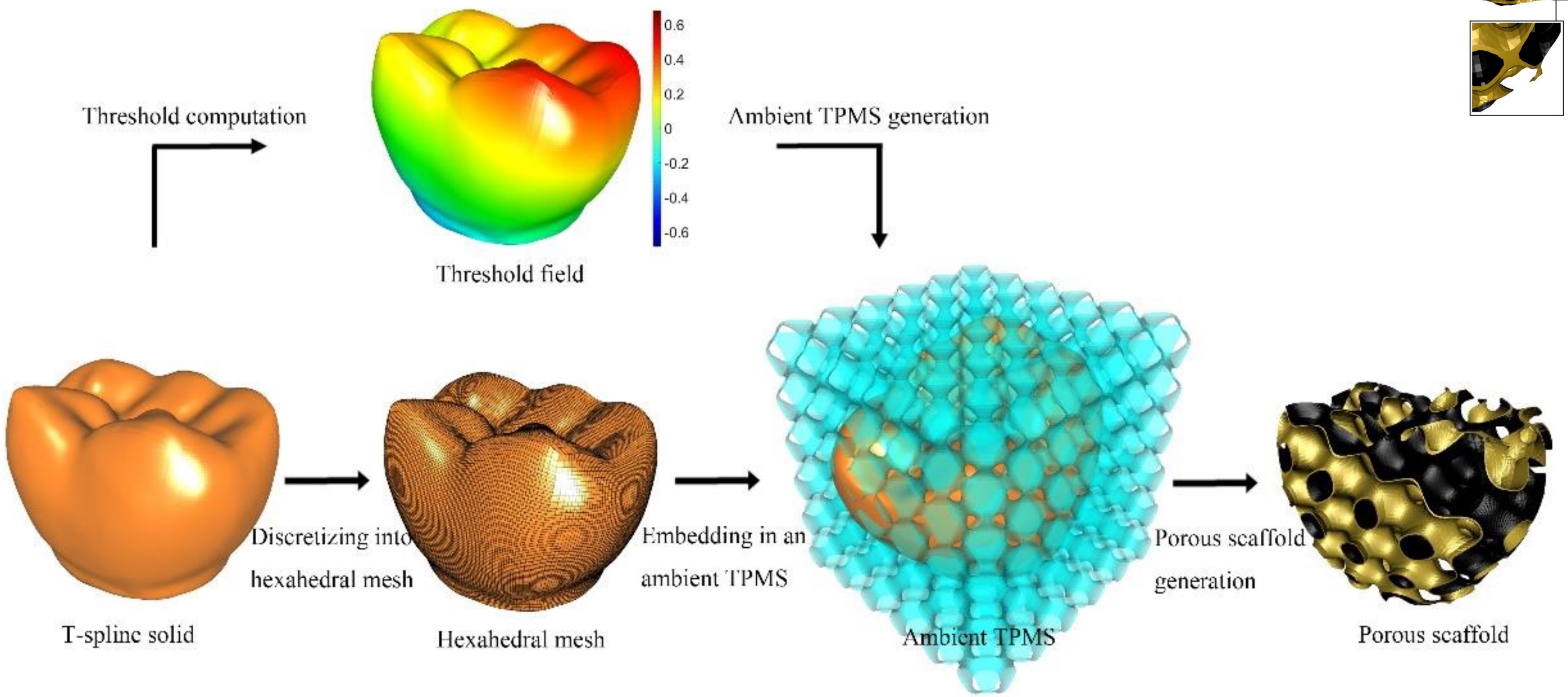
Zitong He, Peisheng Zhuo, Hongwei Lin, Junfei Dai. 3D shape descriptor design based on HKS and persistent homology with stability analysis. Computer Aided Geometric Design, 111 (2024) 102326

Heterogeneous porous : Conventional Methods

- Conventional porous generation methods

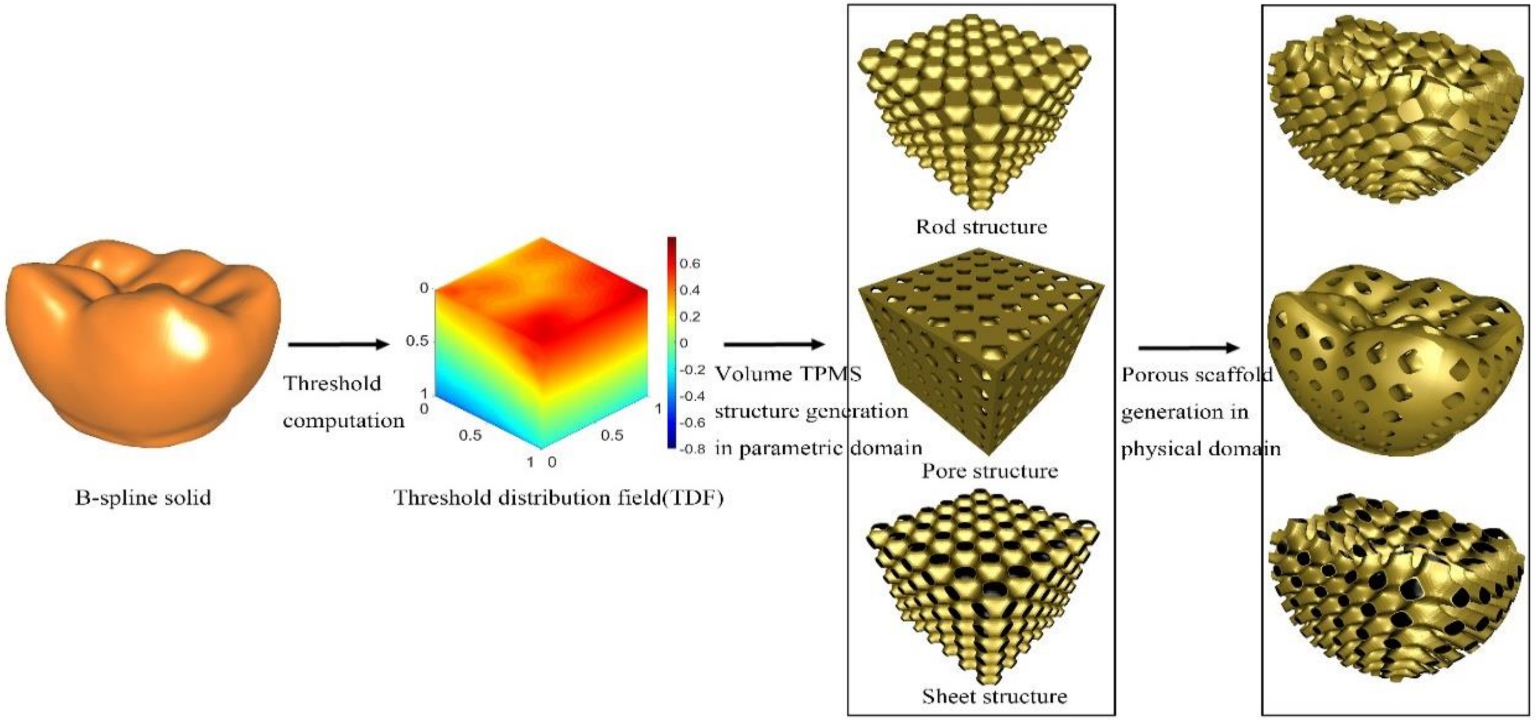
- **Huge storage**
- **Cracked surface**

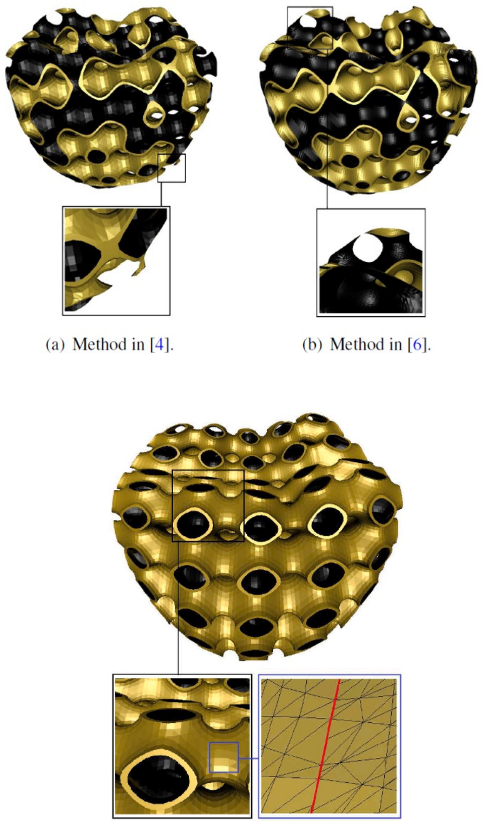




Heterogeneous porous : Our Methods

- Our method: The composition of the free form shape represented by tri-variate B-spline and the implicit B-spline in the parametric domain
- Save at least 99% storage
- Surface integrity





Chuanfeng Hu, Hongwei Lin. Heterogeneous porous scaffold generation using trivariate B-spline solids and triply periodic minimal surfaces. Graphical Model. 115: 101105, 2021

Multilevel porous structure: Algorithm flow

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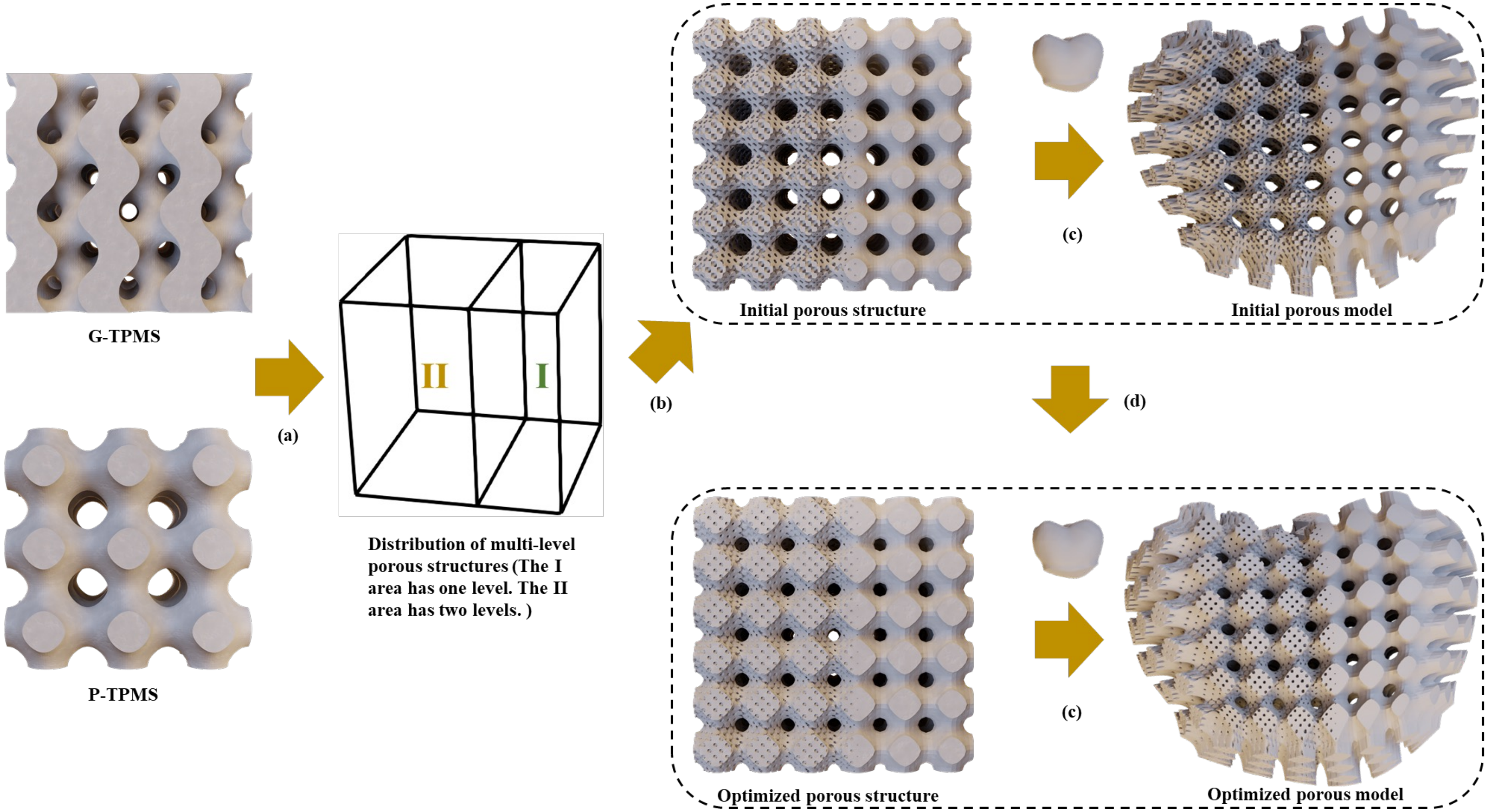
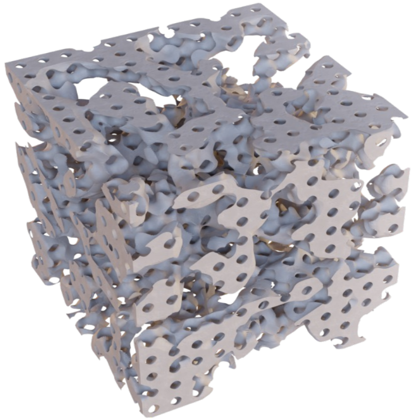
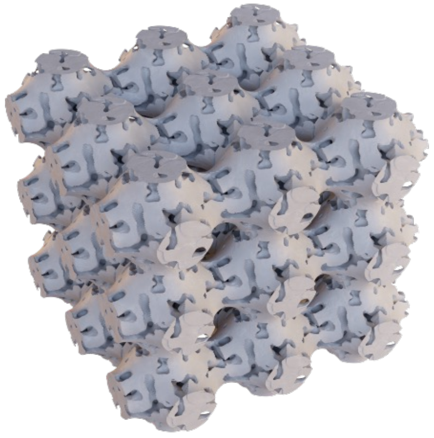
- Represent multilevel porous structure by hierarchical B-spline
- Implicit B-spline is employed to represent porous at each level parametric domain





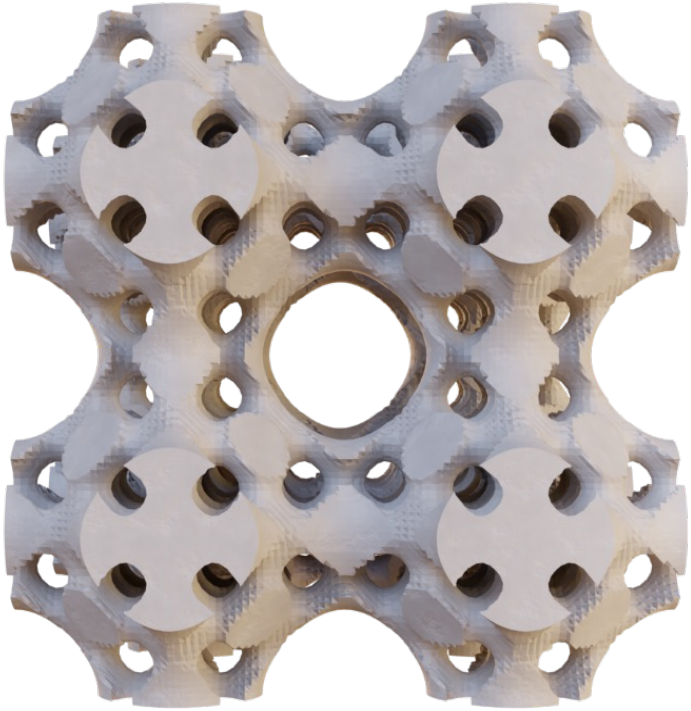
Bionic porous in level-0 and P-TPMS in level-1.



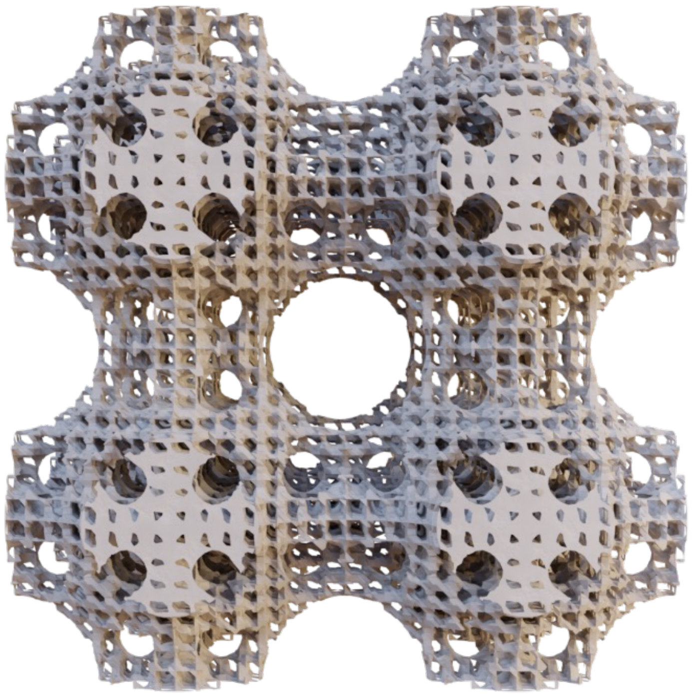
P-TPMS in level-0 and Bionic porous in level-1.

Depeng Gao, Hongwei Lin, Zibin Li. Free-form multi-level porous model design based on truncated hierarchical B-spline functions. Computer-Aided Design, 2023, 162: 103549

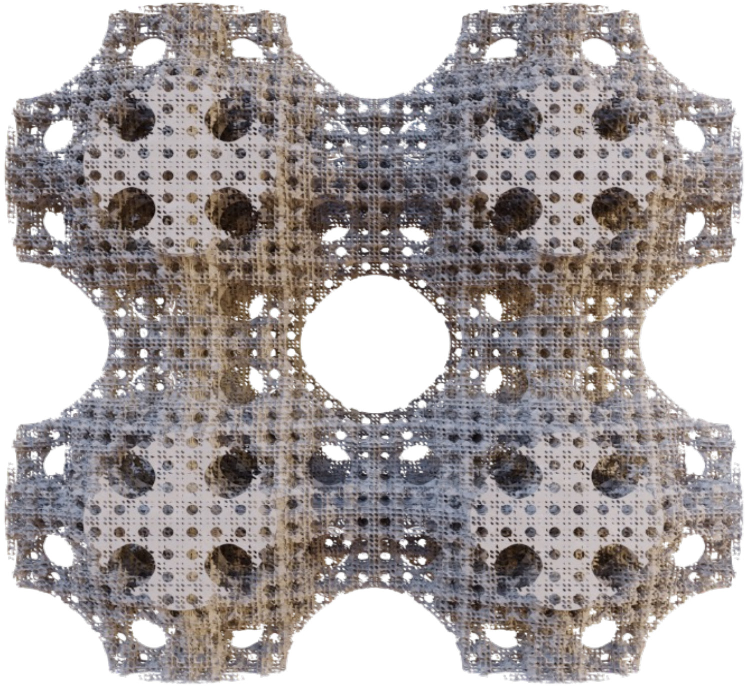
Multilevel porous structure: Examples



Two level



Three level

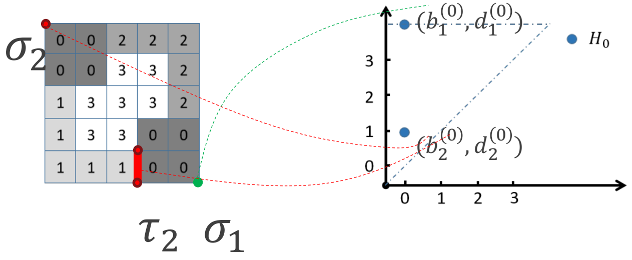


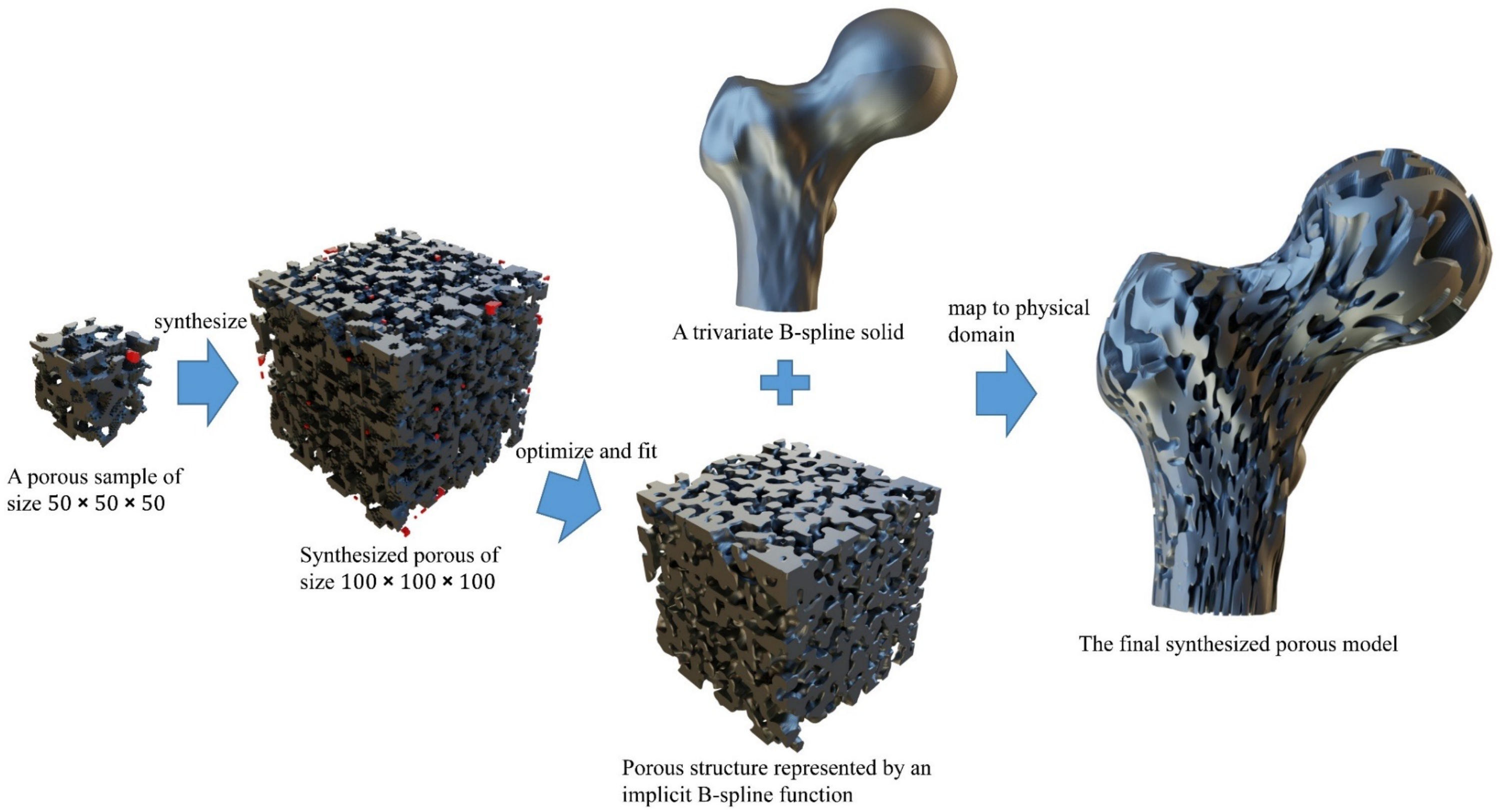
Four level

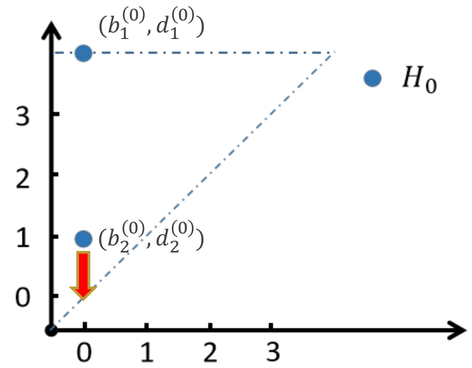
Depeng Gao, Hongwei Lin, Zibin Li. Free-form multi-level porous model design based on truncated hierarchical B-spline functions. Computer-Aided Design, 2023, 162: 103549

Connectivity-guaranteed porous synthesis

- Synthesize the porous in a large region from a small region porous sample using texture synthesis method
- Guarantee the connectivity of the synthesized porous by optimizing the PD





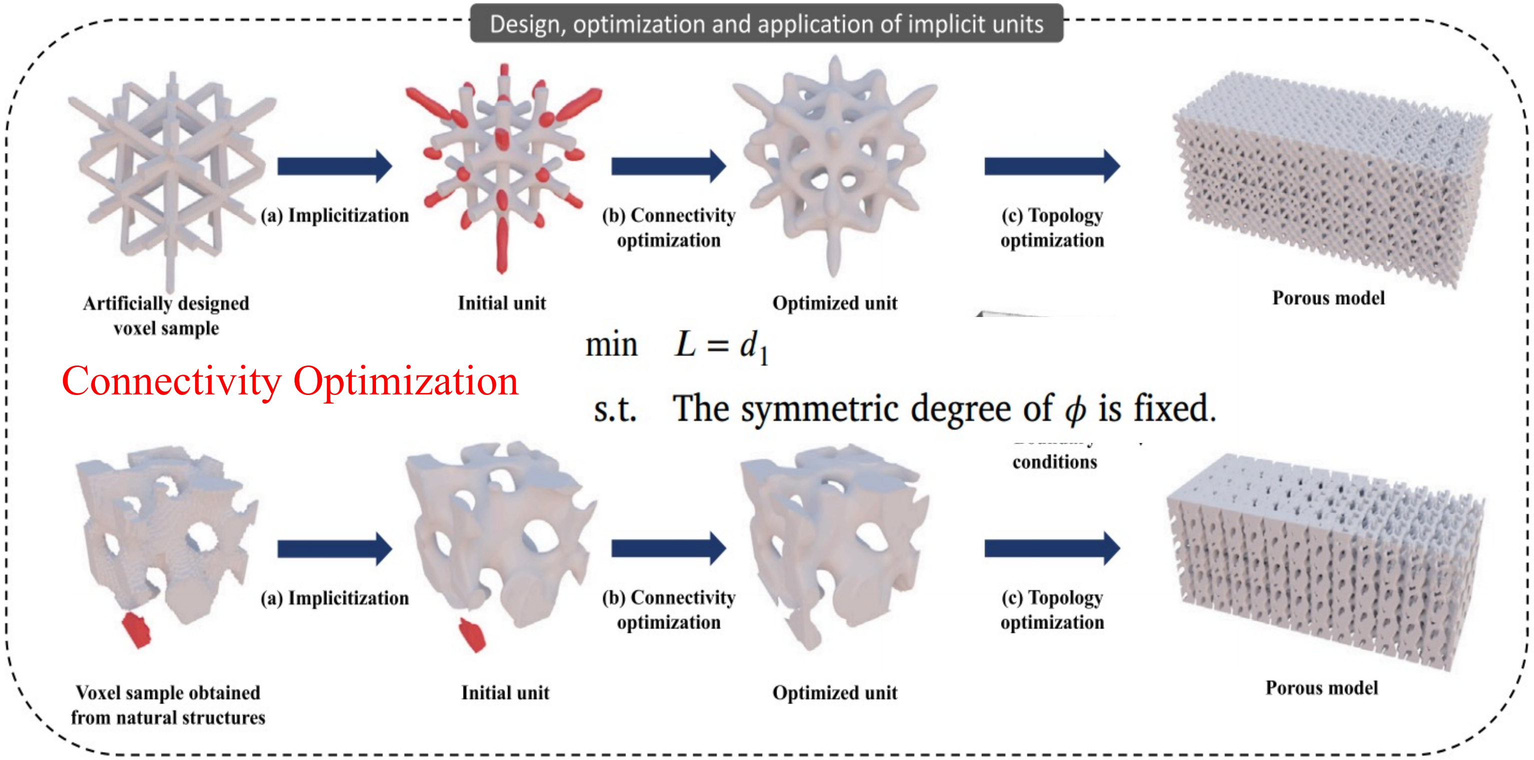


- DTM based filtration
- PD optimization

Depeng Gao, Jinhao Chen, Zhetong Dong, Hongwei Lin. Connectivity-guaranteed porous synthesis in free form model by persistent homology. Computers & Graphics, 106: 33-44 (2022)

Periodic Implicit Representation: Flowchart

- Represent the porous unit by periodic implicit B-spline
- The porous units can be stitched watertight and smoothly
- Enrich the representation forms and design methods of porous structures

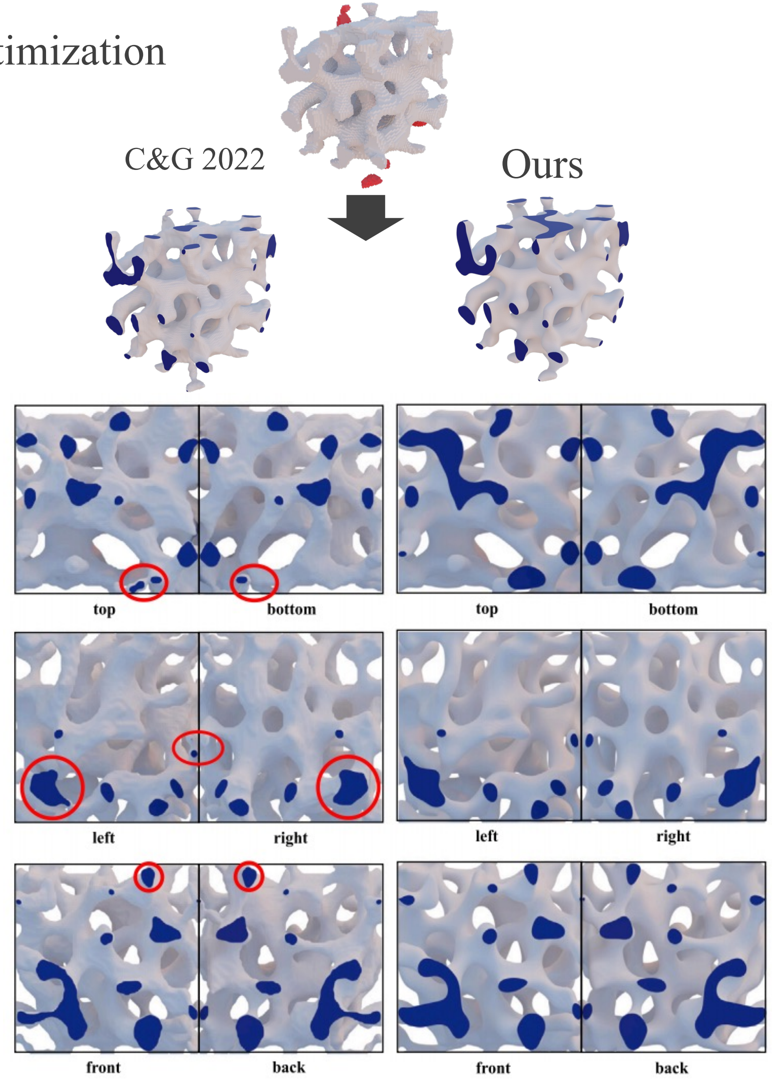


Connectivity Optimization

$$\min \quad L = d_1$$

s.t. The symmetric degree of ϕ is fixed.

Optimization



Depeng Gao, Yang Gao, Hongwei Lin. Periodic implicit representation, design and optimization of porous structure using periodic B-splines. Computer-Aided Design, 171 (2024) 103703

Topology-aware blending method

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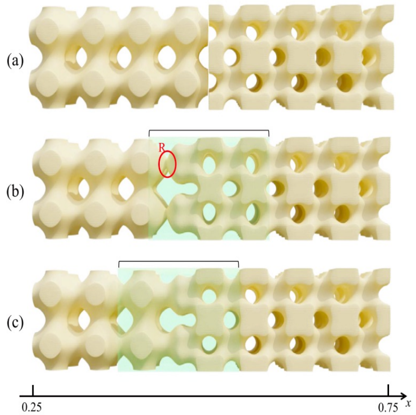
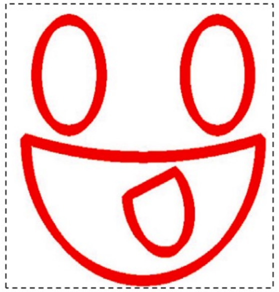
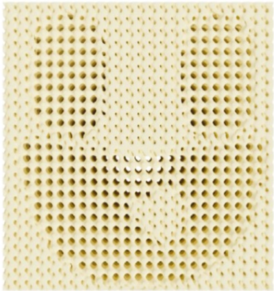
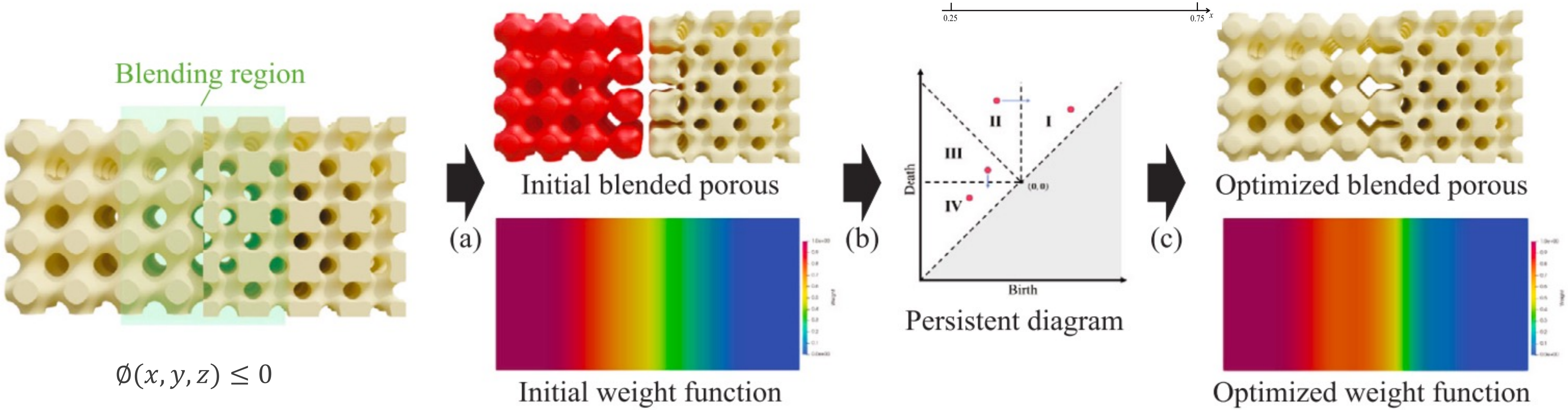
- Blend two implicitly represented porous structures smoothly
- Eliminate the topological errors in the blending region by PD optimization
- Keep the porous outside the blending region unchanged





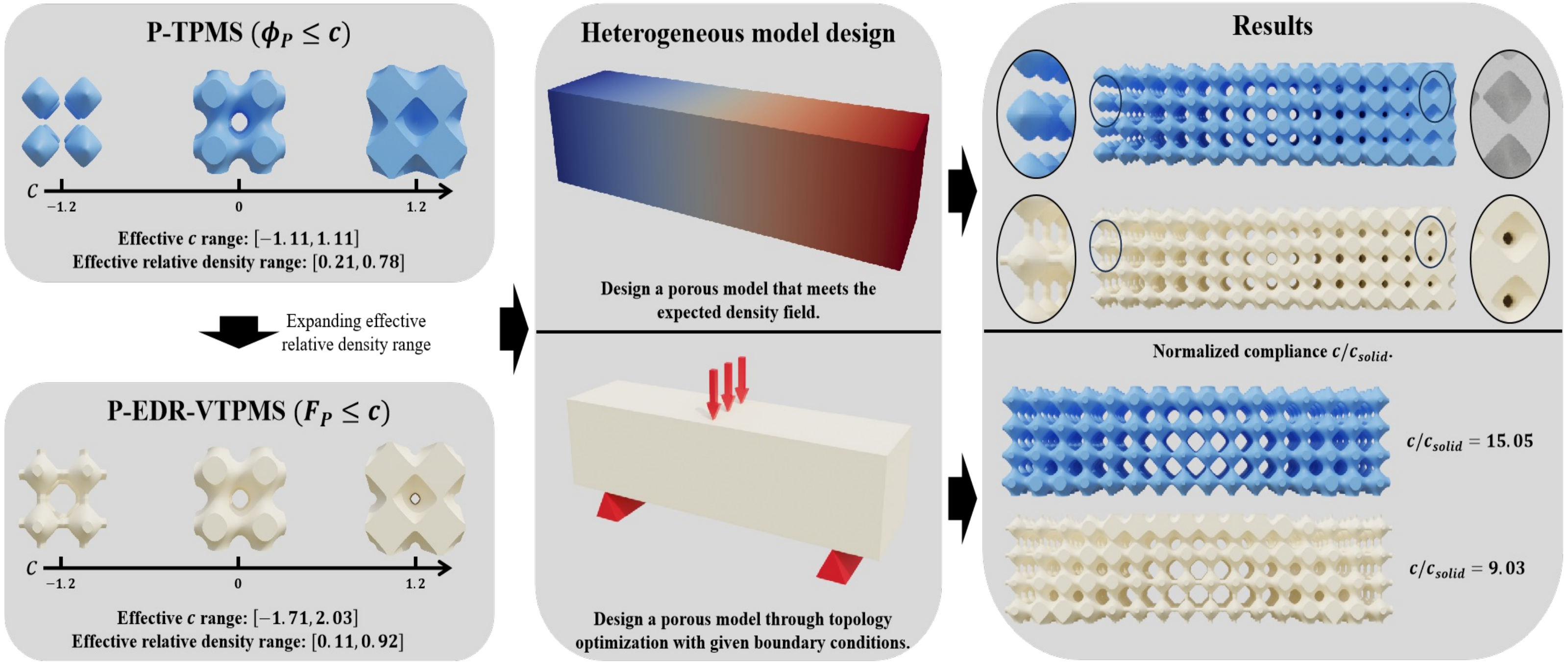


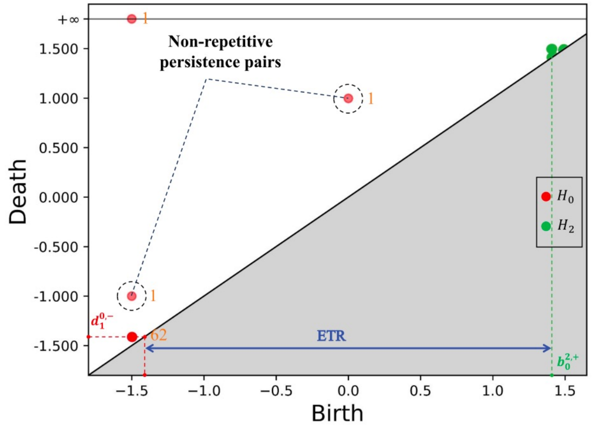


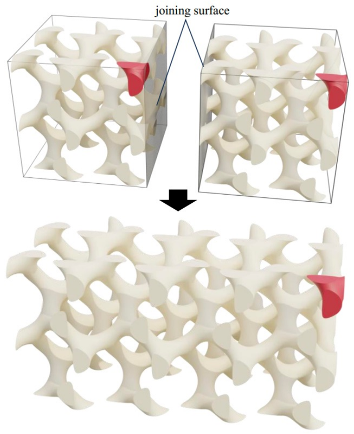
Depeng Gao, Yang Gao, Yuanzhi Zhang, Hongwei Lin. Topology-aware blending method for implicit heterogeneous porous model design. Computer Aided design, 177:103782 (2024)

Persistent Homology-Driven Optimization of Effective Relative Density Range

- Traditional porous representation is TPMS $\Phi(x) < C$
- $\Phi(x)$ is a trigonometric function, with a little adjustable parameters, and limited valid range of C
- Replace $\Phi(x)$ with B-spline， and enlarge the valid range of C by PD optimization





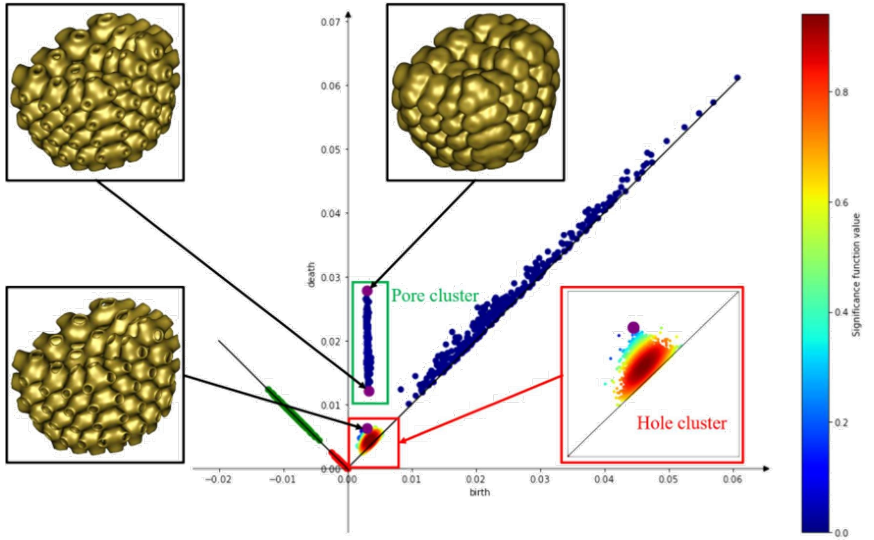


EDR-VTPMS：Effective relative density range-Variant TPMA

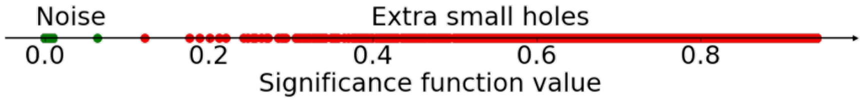
Depeng Gao, Yuanzhi Zhang, Hongwei Lin. Persistent Homology-Driven Optimization of Effective Relative Density Range for Triply Periodic Minimal Surface. Computer-Aided Design, in revision.

Reasonable thickness determination for sheet structure

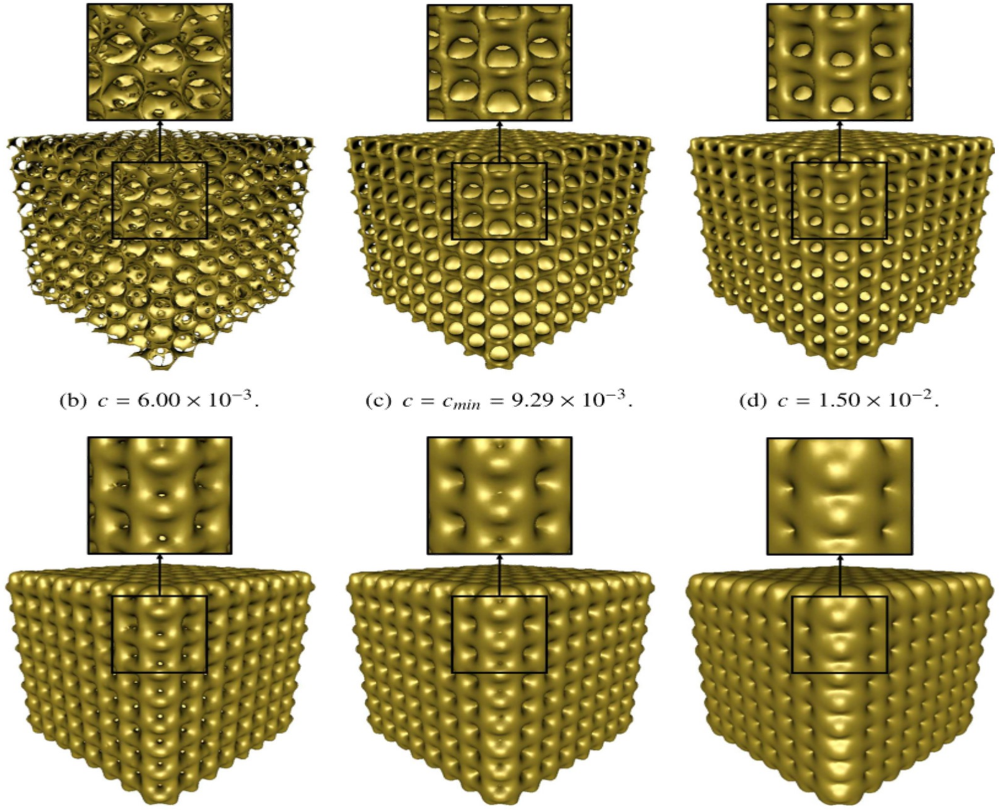
- TPMS surface represented porous should have thickness for 3D
- With too small thickness, there will be extra small holes; with too large thickness, porous will be closed
- By clustering the points in a PD, determine the reasonable thickness range



(a) 1-PD of the scalar field $|f(x, y, z)|$.



(b) Significance function values of the points in the short persistence cluster.



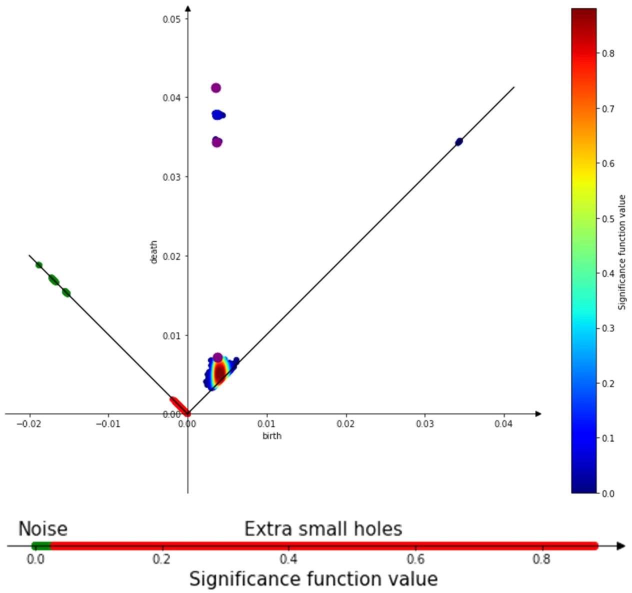
(b) $c = 6.00 \times 10^{-3}$. (c) $c = c_{min} = 9.29 \times 10^{-3}$. (d) $c = 1.50 \times 10^{-2}$.

(e) $c = c_{max,1} = 2.26 \times 10^{-2}$. (f) $c = c_{max,2} = 2.58 \times 10^{-2}$. (g) $c = 4.00 \times 10^{-2}$.

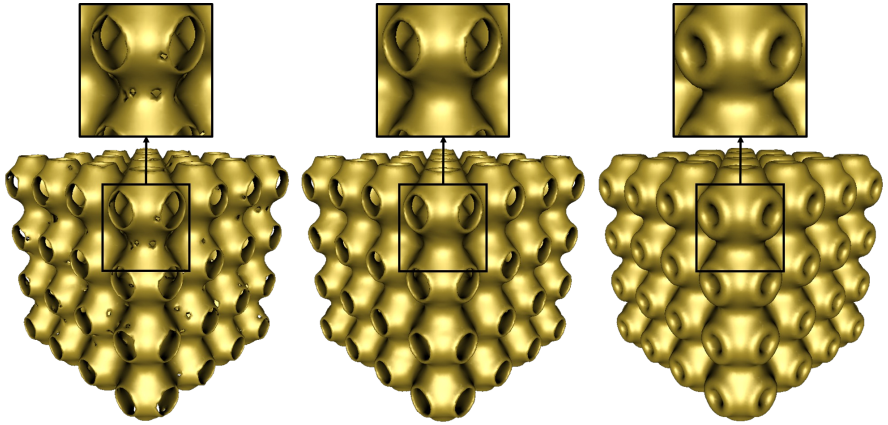
Jiacong Yan, Hongwei Lin. Reasonable thickness determination for implicit porous sheet structure using persistent homology. Computers & Graphics, 115, 236-245, 2023

Reasonable thickness determination for porous sheet structure

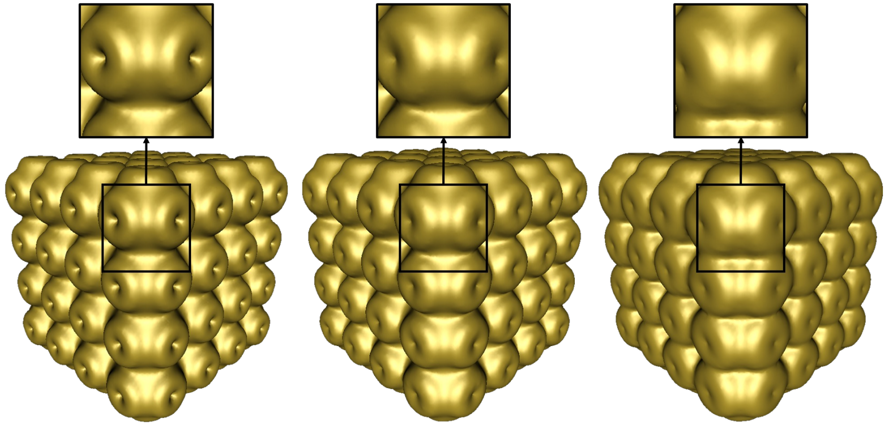
Experimental Results:



(1) 1-PD of scalar field $|f(x,y,z)|$.



(2) $c = 6.00 \times 10^{-3}$. (3) $c = c_{min} = 7.20 \times 10^{-3}$. (4) $c = 2.00 \times 10^{-2}$.



(5) $c = c_{max,1} = 3.43 \times 10^{-2}$. (6) $c = c_{max,2} = 4.13 \times 10^{-2}$. (7) $c = 6.00 \times 10^{-2}$.

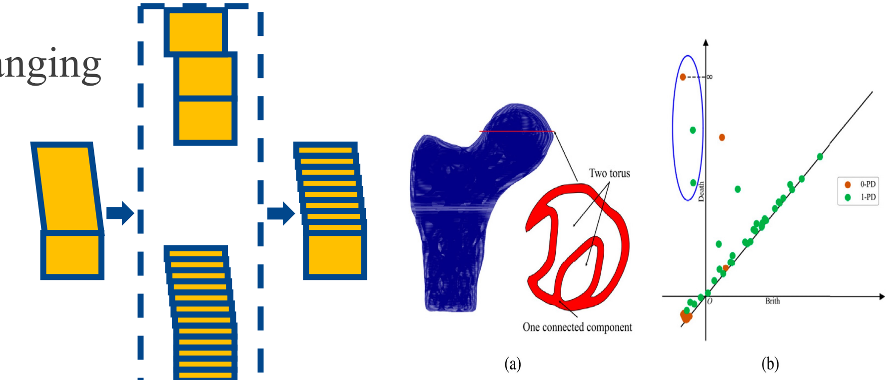
The variation of the number of pores and holes in these sheet structures

Parameter c	Topological measurement $\bar{\beta}$
$c < c_{min}$	5.661
$c = c_{min}$	**1.000**
$c_{min} < c < c_{max,1}$	**1.000**
$c = c_{max,1}$	0.998
$c = c_{max,2}$	0.000
$c > c_{max,2}$	0.000

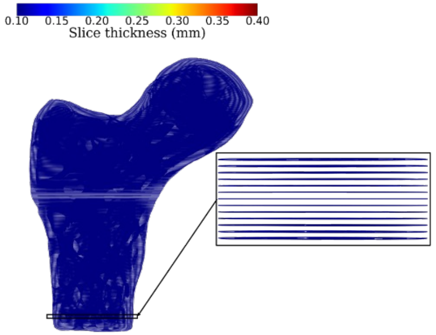
Jiacong Yan, Hongwei Lin. Reasonable thickness determination for implicit porous sheet structure using persistent homology. Computers & Graphics, 115, 236-245, 2023

Adaptive slicing with topology guarantee

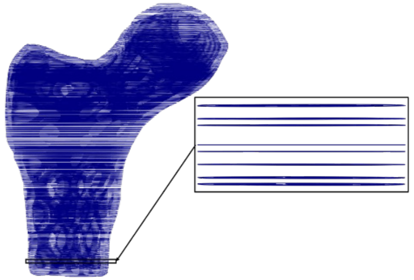
- The 2D slice of a porous has complicated topological structure
- By comparing the PDs of adjacent slices, find the critical slice with topology changing
- Guarantee the topology consistency between design model and printed model



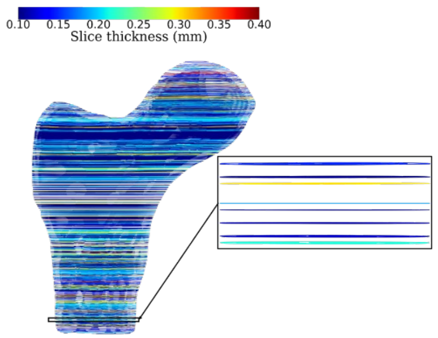
Input implicit porous structure $f(x, y, z) \leq 0$



Finest slice model generation with the minimum thickness h_{min}



Slices for printing generation



Slice thicknesses optimization

Model	Method	Slice number	Topology detection error	Cusp height sum (mm)	Volume deviation
Balljoint	**Our method**	502	**0**	**44.34**	**0.31%**
	Cusp height based method	502	192	47.52	0.39%
	Boolean operations based method	502	197	46.34	0.40%
Moai	**Our method**	580	**0**	**35.56**	**0.40%**
	Cusp height based method	580	192	37.45	0.41%
	Boolean operations based method	580	148	37.02	0.41%
Tooth	**Our method**	652	**0**	**54.63**	**0.58%**
	Cusp height based method	652	274	56.84	0.62%
	Boolean operations based method	652	233	55.93	0.62%
Venus	**Our method**	569	**0**	**51.31**	**0.55%**
	Cusp height based method	569	227	54.23	0.60%
	Boolean operations based method	569	264	53.20	0.61%

Jiacong Yan, Hongwei Lin. Adaptive Slicing of Implicit Porous Structure with Topology Guarantee. Computer-Aided Design, 162, 103557, 2023 (SPM 2023).

目录 CONTENTS

Functional Network

- Neural networks have a close relationship between their structures and functions
- We propose the concept of the functional network of neural networks
- By exploring the topological structure of these functional networks, we can gain a better understanding of the connection between their structures and functions

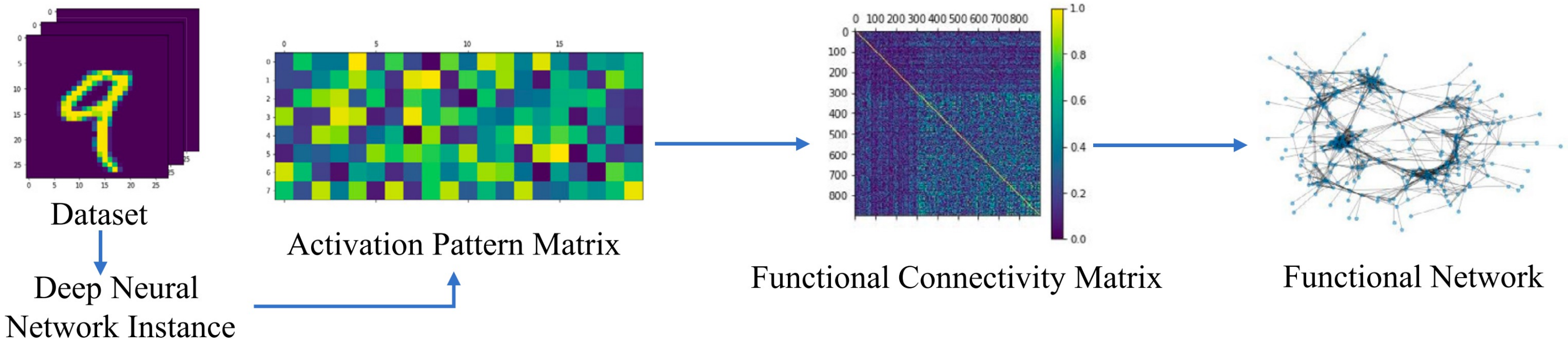
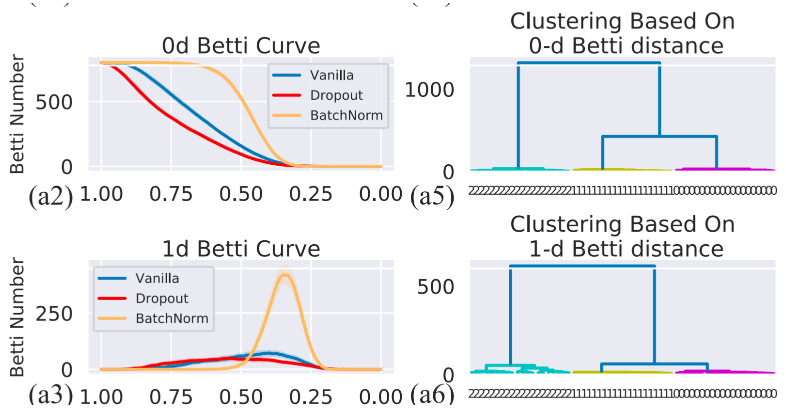
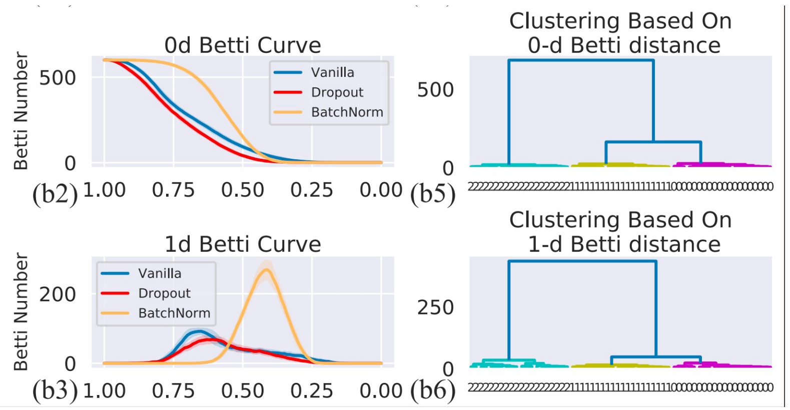


Fig. 1. Flow chart of constructing the functional network for a given deep neural network.



Fashion MNIST [300, 300]



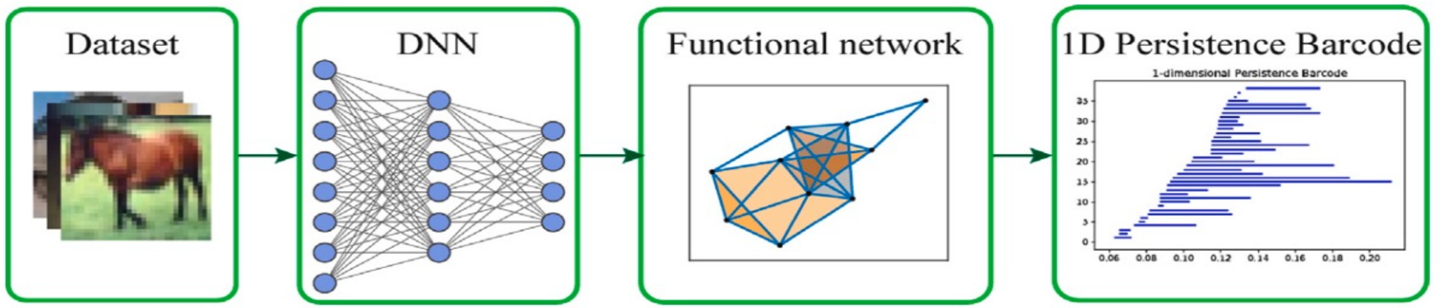
MNIST [300, 300]

Influence of different regularizations

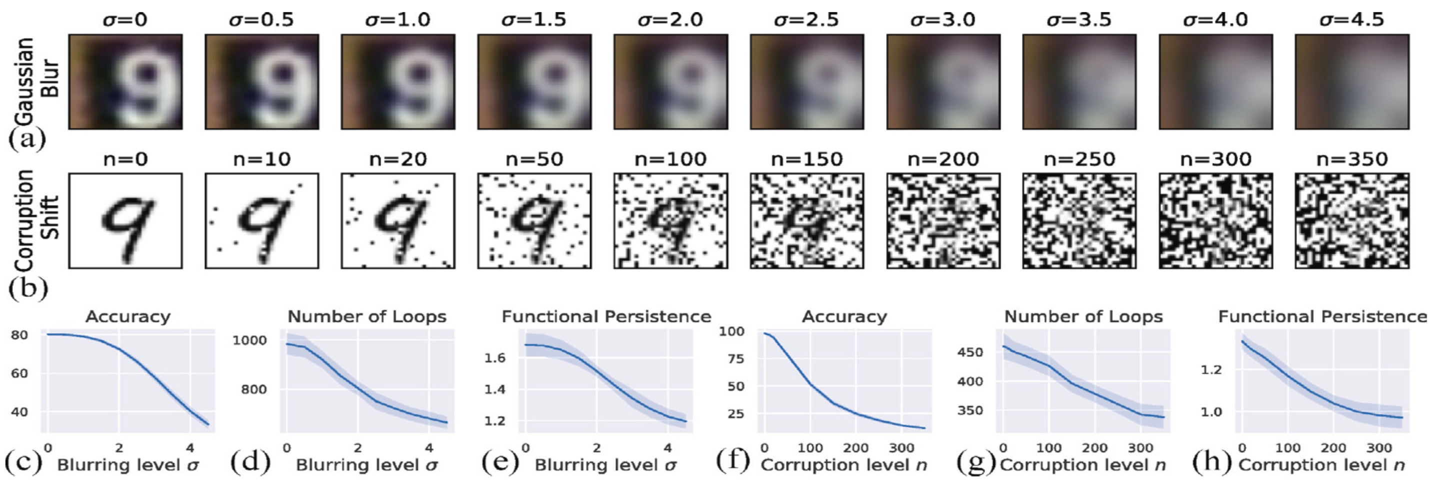
Ben Zhang, Zhetong Dong, Junsong Zhang, Hongwei Lin. Functional network: A novel framework for interpretability of deep neural networks. Neurocomputing 519: 94-103 (2023)

Functional Loops

- The relationship between functional organizations and network performance is explored using algebraic topology
- The functional loops reveal functional interaction patterns of multiple neurons in DNNs.



- The functional network is constructed as a simplicial complex $K(M, S)$ by computing the functional similarities between neurons
- The 1-dimensional persistent barcode is calculated to measure the functional complexity of the DNN



- The number of loops, functional persistence, and accuracy decrease as the dataset shifts
- Therefore, the number of loops may reflect the number of features extracted by models from unseen samples

Ben Zhang, Hongwei Lin. Functional loops: Monitoring functional organization of deep neural networks using algebraic topology. Neural Networks 174: 106239 (2024)

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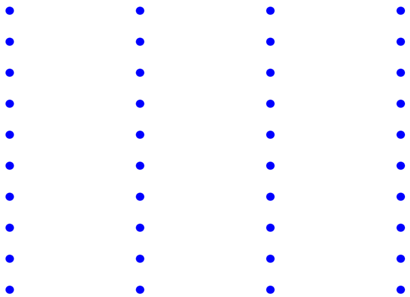
Computational Psychology and Gestalt Computational Model

- Computational psychology studies computational models of psychological rules
- The Gestalt theory is a classic theory in cognitive psychology used to explain the role of visual perception
- We established the Gestalt computational model using persistent homology theory in computational topology
- Gestalt theory includes five core principles: similarity, proximity, closure, good continuation, and pragnanz

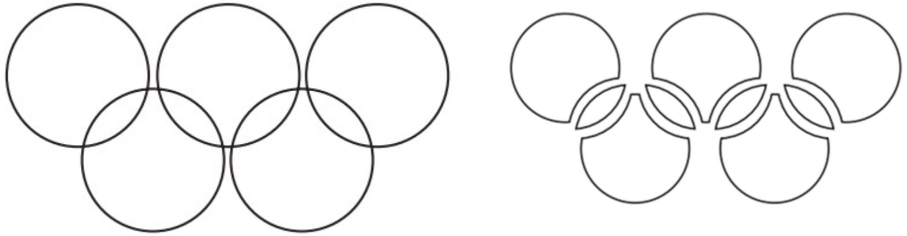
Similarity



Proximity

Closure

Good continuation



Pragnanz

Gestalt Computational Model-Step 1,2,3

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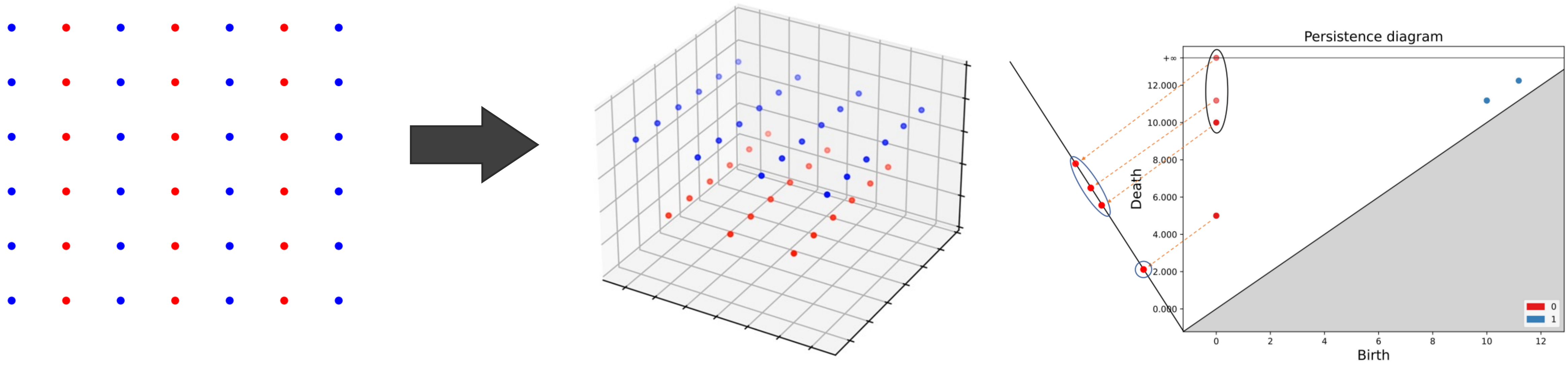
1. **Set additional coordinates**: According to the attributes of points set that affect the visual perception, assign an additional z-coordinate,

$$\{Q_i = (x_i, y_i, z_{i,1}, z_{i,2}, \cdots, z_{i,m}), i = 1,2,\cdots,n\}$$

2. **Calculate PD:** Construct VR filtration and calculate the corresponding PD

3. **Cluster the points on the PD:**

   - Project the points in the PD onto the line y = -x and cluster them into two categories
   - The category far from the origin is the important feature points



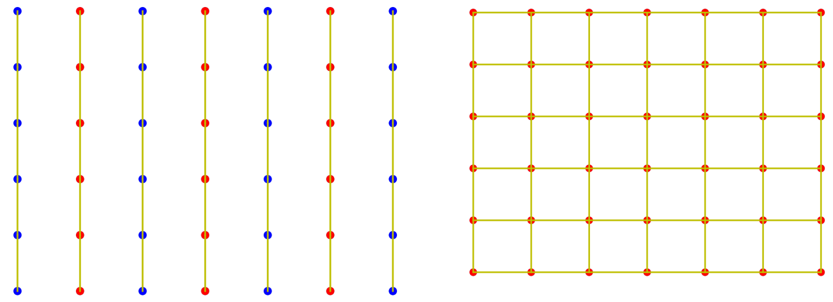
Gestalt Computational Model-Step 4, 5

4. Determine the threshold: Determine a suitable threshold ε_g so that important topological features have appeared and not disappeared

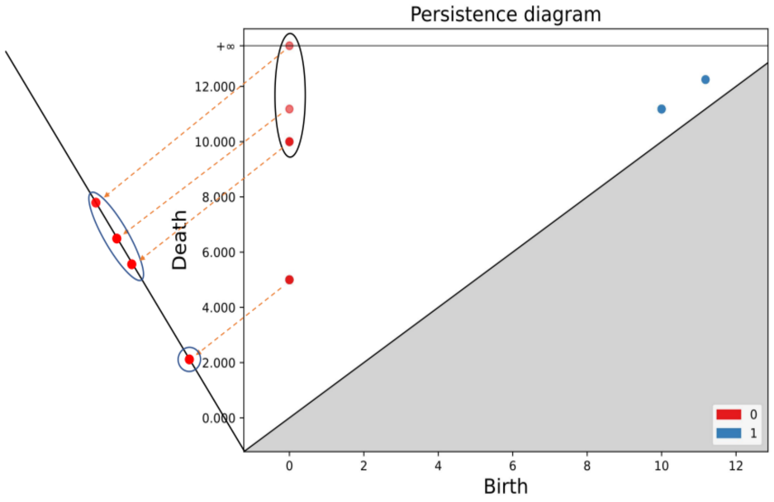
- For 0-PD, select ε_g as the maximum value of the extinction values of all noise points
- For 1-PD, select ε_g as the maximum value of the birth values of all important points

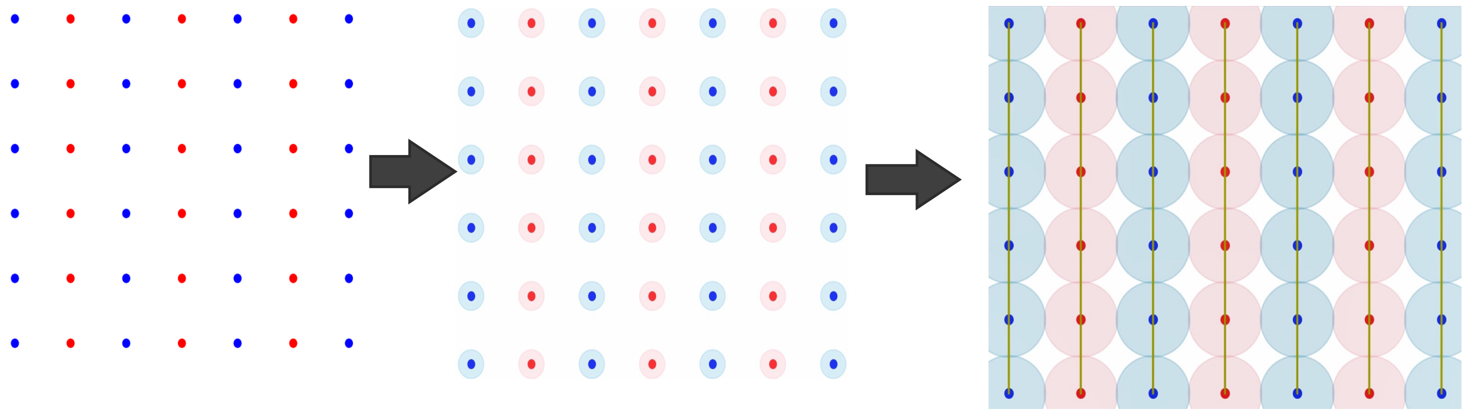
5. Reconstruction of visual perception results:

- Reconstruct the results of visual perception from $VR(X, \varepsilon_g)$

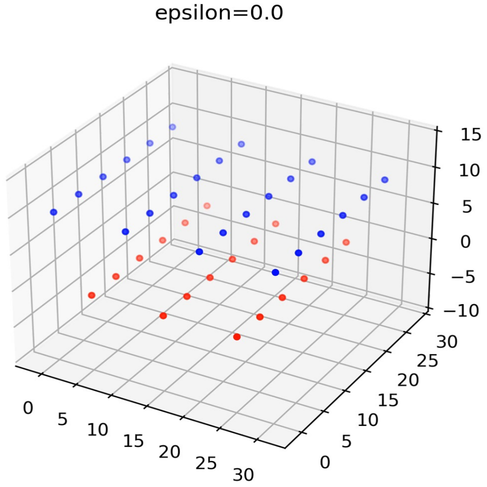


The visual perception results reconstructed by the above method are in line with Gestalt principles

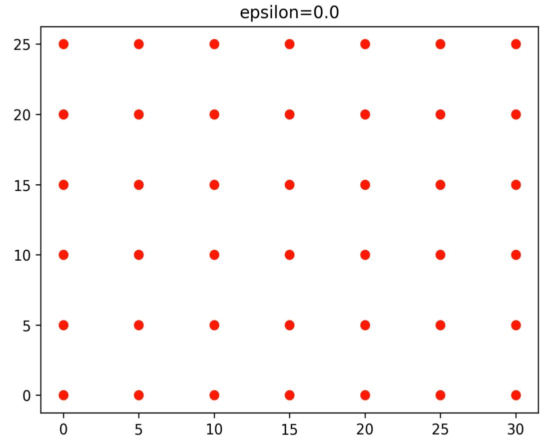




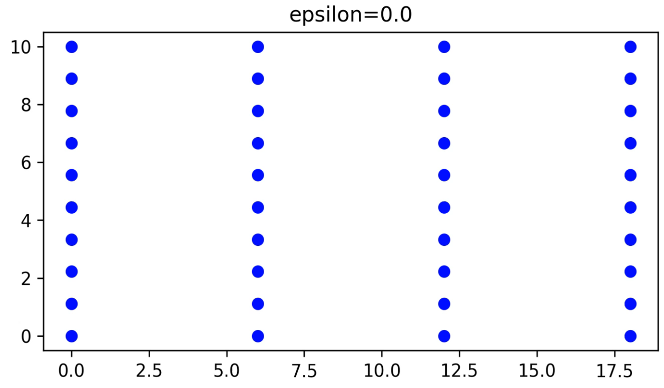
Gestalt Computational Model: Results



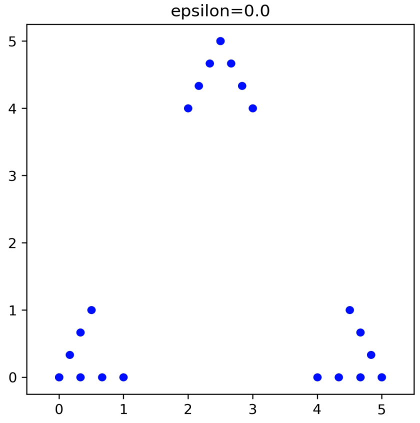
Similarity: 3D



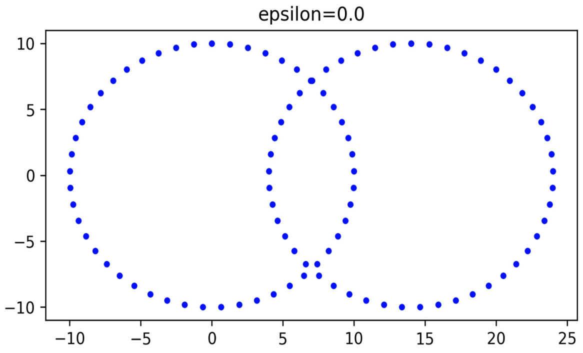
Similarity: 2D



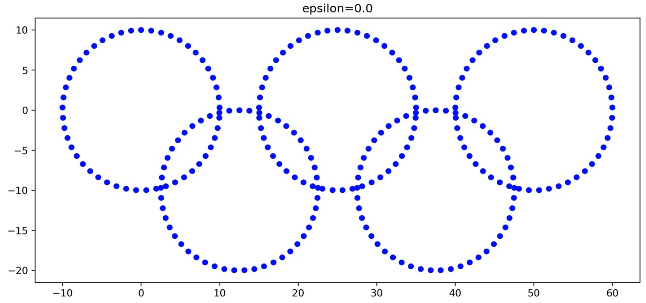
Proximity



Closure

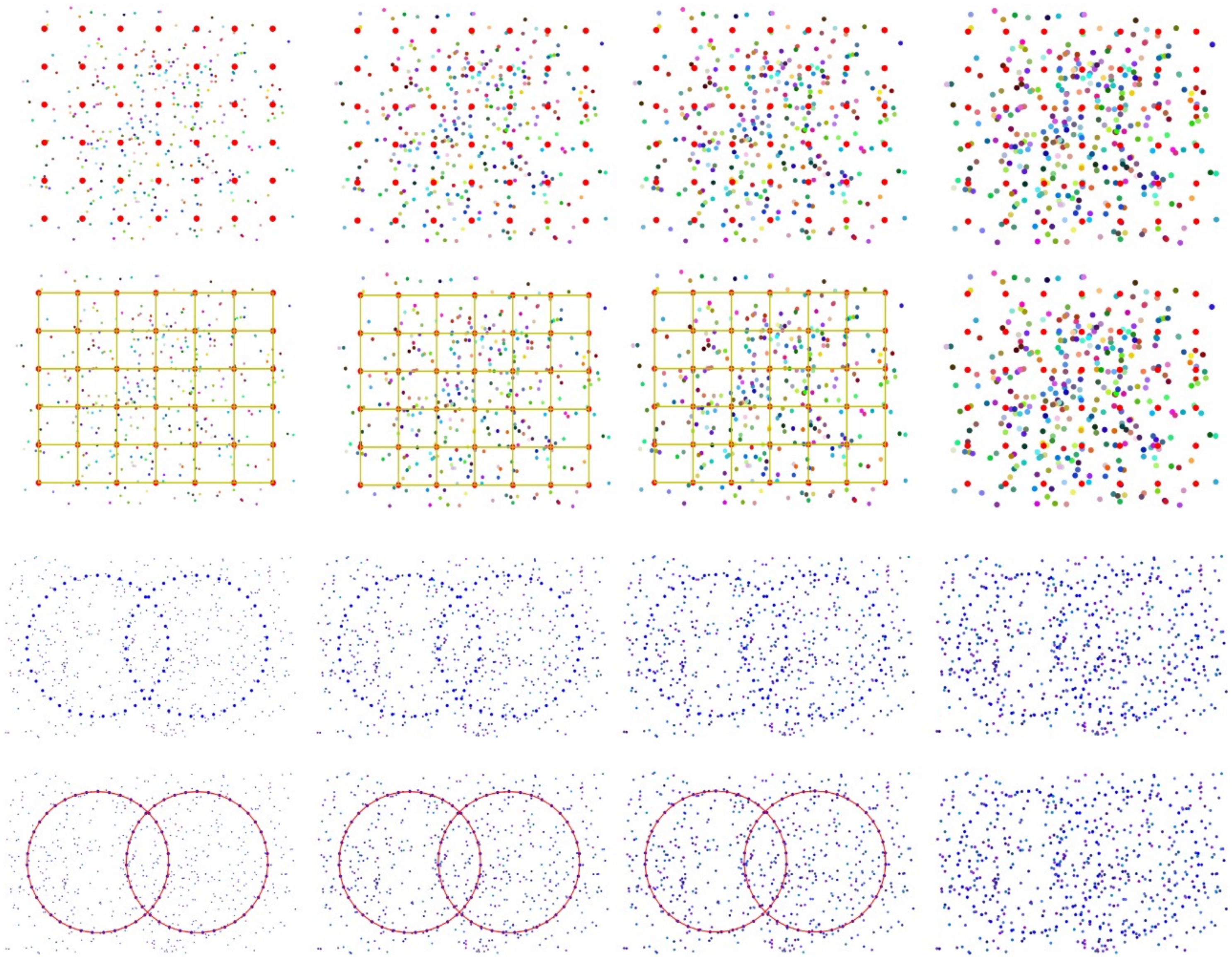


Good continuation

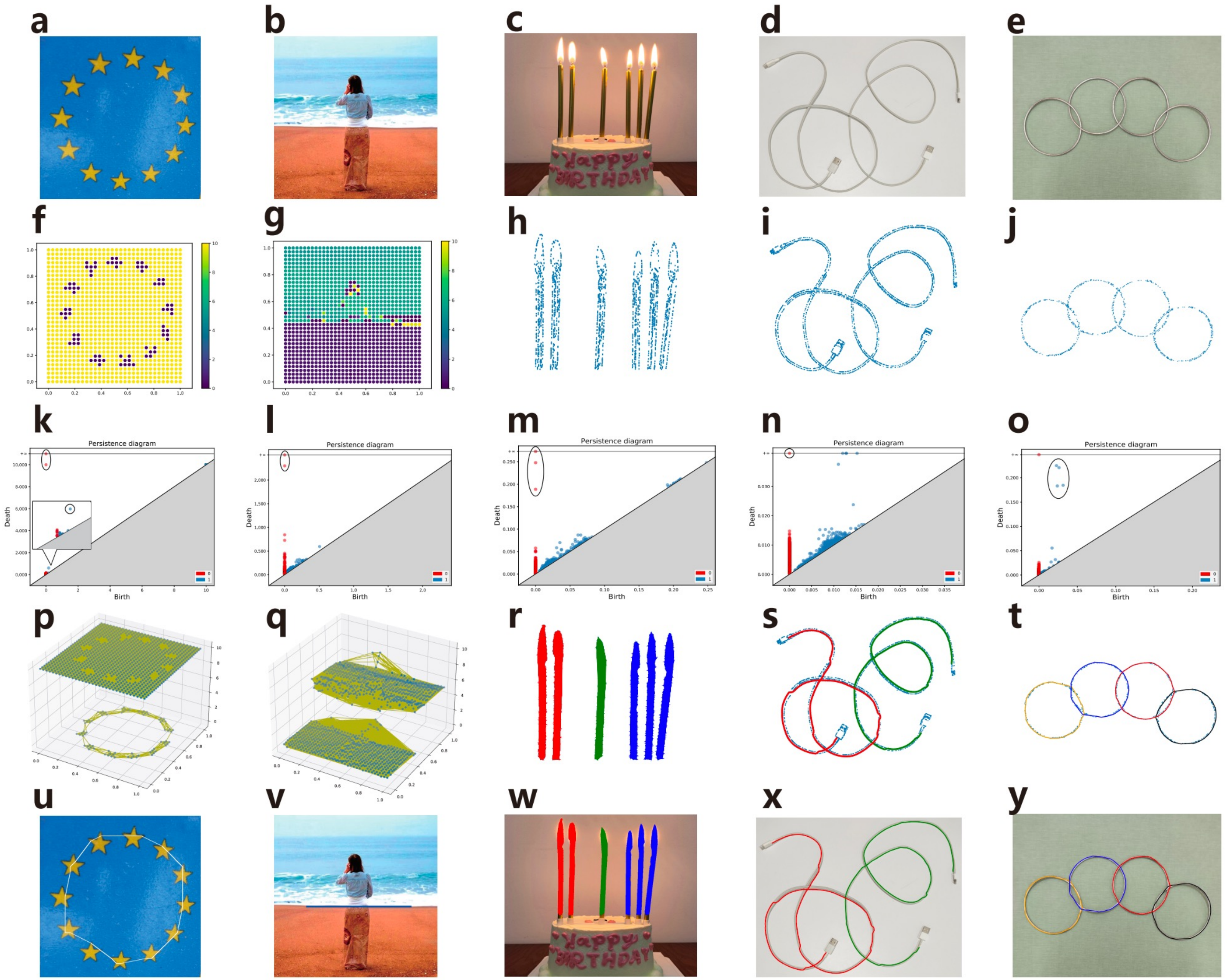


Pragnanz

Influence of noise



Computational Results on Real Images



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Conclusion

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- Computer Aided Geometric Design →Computer Aided Topological Design
- 计算机辅助拓扑设计：以持续同调为主要理论工具，系统解决几何设计和几何处理中的拓扑问题

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计算机辅助拓扑设计

——持续同调在几何设计和处理中的应用

董哲同[1,2]， 蔺宏伟[1,2]

(1. 浙江大学数学科学学院，浙江 杭州 310027；
2. 浙江大学 CAD&CG 国家重点实验室，浙江 杭州 310058)

摘 要：持续同调是一种计算不同尺度拓扑特征的有效方法。其从一簇向后包含的单纯复形序列中提取出拓扑特征的出现和消失时刻，并使用拓扑特征的“生命周期”来量化地衡量该特征的几何尺度和重要程度。拓扑特征的提取与应用在几何设计中扮演着重要角色，催生出了一些基于持续同调的几何设计研究。从持续同调特征的提取与基于持续同调的建模和优化两方面进行综述，在持续同调特征的提取方面，介绍了从点云和三角网格数据中提取拓扑特征的不同方法，总结了拓扑特征在部分几何设计问题中的应用路径。在建模和优化方面，综述了基于拓扑变换的单纯复形重建方法、拓扑可感知的曲面重建方法与基于持续同调的拓扑去噪和优化方法。

关 键 词：持续同调；拓扑特征提取；形状重建；去噪与优化；几何设计；拓扑设计
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Computer aided topological design

——survey on geometric design and processing based on persistent homology

DONG Zhe-tong[1,2], LIN Hong-wei[1,2]

(1. School of Mathematical Sciences, Zhejiang University, Hangzhou Zhejiang 310027, China;
2. State Key Laboratory of CAD&CG, Zhejiang University, Hangzhou Zhejiang 310058, China)

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